Chapter 4 Microwave transmission lines

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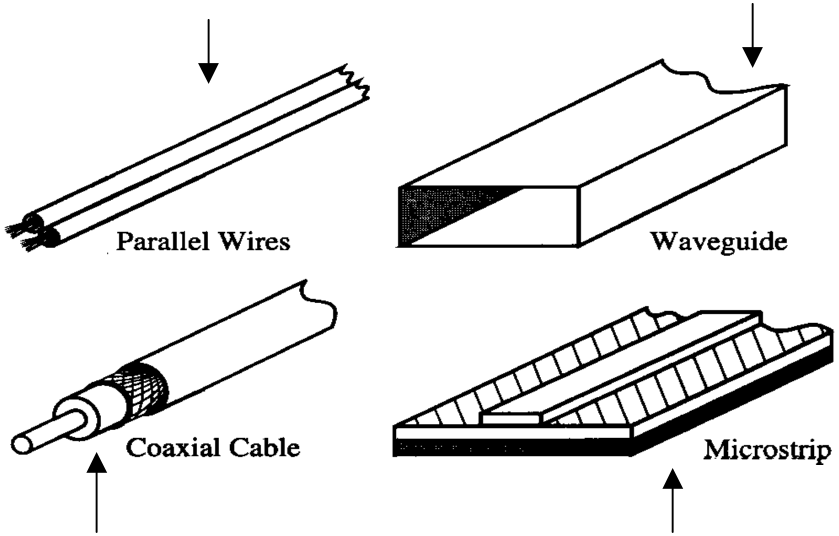
4.1 Introduction

1. Microwave transmission lines

low frequency use,
high radiation loss

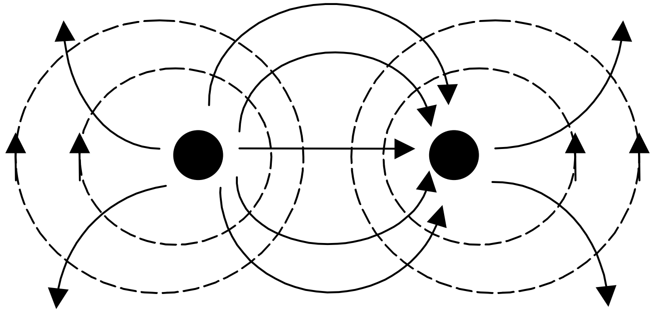
good all-around characteristics,
but difficult in integration with
other microwave components

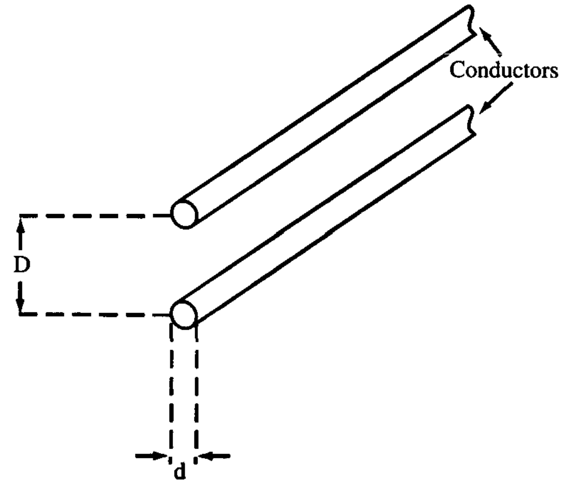


wideband TEM operation,
low power handling capability

well suited for MMIC(monolithic microwave integrated circuit) or HMIC(hybrid microwave integrated circuit)

4.2 Two-wire lines





1. Dominant mode: TEM mode

$$C=\frac{\pi\varepsilon'}{\cosh^{-1}D/d}\approx\frac{\pi\varepsilon'}{\ln 2D/d}, L=\frac{\mu}{\pi}\cosh^{-1}\frac{D}{d}\approx\frac{\mu}{\pi}\ln\frac{2D}{d}\ (\text{for D >> d})$$

$$R=\frac{R_s}{\pi d/2}, R_s=\sqrt{\frac{w\mu}{2\sigma_c}},\ G=\frac{\pi w\varepsilon''}{\cosh^{-1}D/d}=\frac{\sigma_d}{\varepsilon'}C, \varepsilon=\varepsilon'-j\varepsilon''=\varepsilon'-j\frac{\sigma_d}{w}$$

$$Z_o=\sqrt{\frac{L}{C}}=\frac{1}{\pi}\sqrt{\frac{\mu}{\varepsilon}}\ln\frac{2D}{d}=\frac{120}{\sqrt{\varepsilon_r}}\ln\frac{2D}{d}=\frac{276}{\sqrt{\varepsilon_r}}\log\frac{2D}{d}$$

2.Attenuation constant for a low loss line

$$\gamma = \alpha + j\beta = \sqrt{(R + jwL)(G + jwc)}$$

$$\approx jw\sqrt{LC}[1 - \frac{j}{2}(\frac{R}{wL} + \frac{G}{wC})]$$

$$\rightarrow \alpha \approx \frac{1}{2}(\frac{R}{Z_o} + GZ_o) = \alpha_c + \alpha_d , \beta = w\sqrt{LC}$$

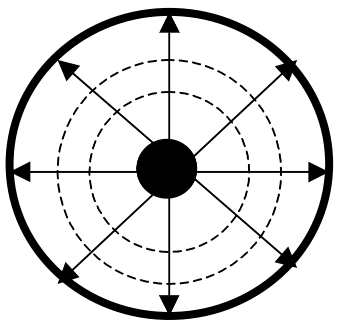
3. Ex. 4.1 2-wire line with d=0.2cm, D=0.8cm → Z_o=249.3Ω
4. Operation frequency < 500MHz, e.g., TV twin-lead, Z_o usually ranges from 150 Ω to 600 Ω.

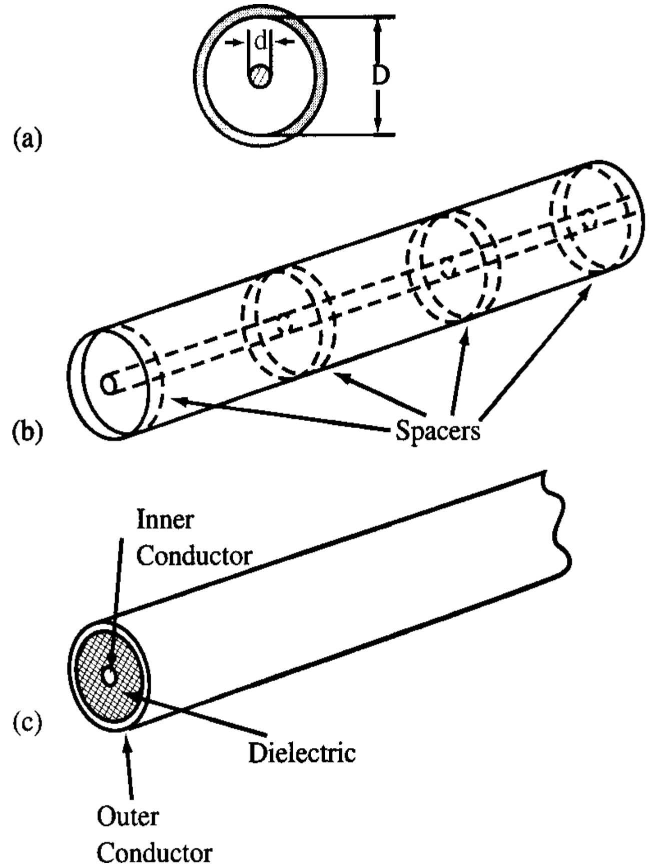
4.3 Coaxial line

1. Dielectric material

Dielectric material	ε_r	$\tan\delta$
Teflon	2.08	0.0004
Polyethylene	2.25	0.0004

2. Dominant mode: TEM mode





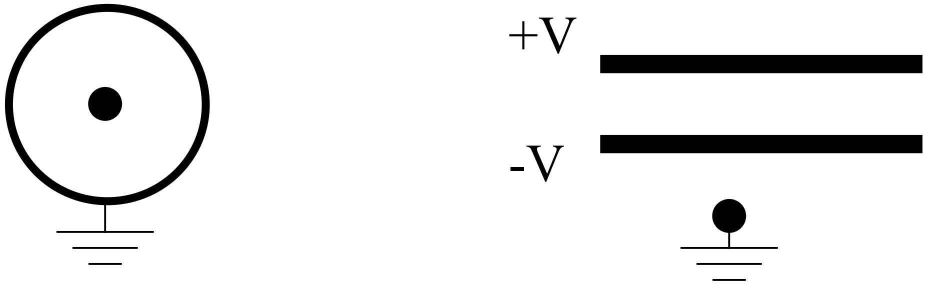
$$C = \frac{2\pi\varepsilon'}{\ln D/d}, L = \frac{\mu}{2\pi}\ln\frac{D}{d}$$

$$R = \frac{R_s}{\pi}\left(\frac{1}{d}+\frac{1}{D}\right),\ G = \frac{2\pi w\varepsilon''}{\ln D/d}$$

$$Z_o = \sqrt{\frac{L}{C}} = \frac{1}{2\pi}\sqrt{\frac{\mu}{\varepsilon}}\ln\frac{D}{d} = \frac{60}{\sqrt{\varepsilon_r}}\ln\frac{D}{d} = \frac{138}{\sqrt{\varepsilon_r}}\log\frac{D}{d}$$

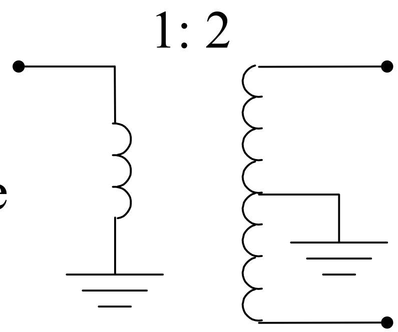
3. Ex.4.2 a coaxial line with D=7/8in, d=1/4in, ε_r=2.25→Zo=50Ω

4. unbalanced line:one conductor is at ground potential
   balanced line: the voltage to ground of the two conductors are equal and opposite



5. balun: a device to provide a low VSWR transition between a *bal*anced one to *un*balanced one.

unbalanced line
75Ω coaxial line



balanced line
300Ω TV twin-lead

6. Power capacity is limited by the voltage breakdown E_d (=3×10^6V/m for room temperature air)
an air-filled coaxial line

$$E_\rho = \frac{V_o}{\rho \ln \frac{D}{d}} \rightarrow V_{\max} = \frac{d}{2} E_d \ln\frac{D}{d} \rightarrow P_{\max} = \frac{1}{2}\frac{V_{\max}^2}{Z_o} = \frac{\pi (d/2)^2 E_d^2}{\eta_o} \ln\frac{D}{d}$$

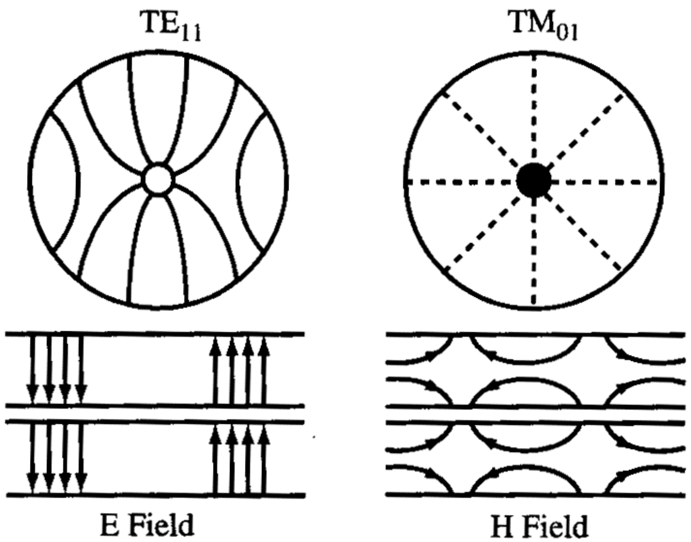
Ex. RG-142 cable size D/2=0.114in, d/2=0.035in, using air

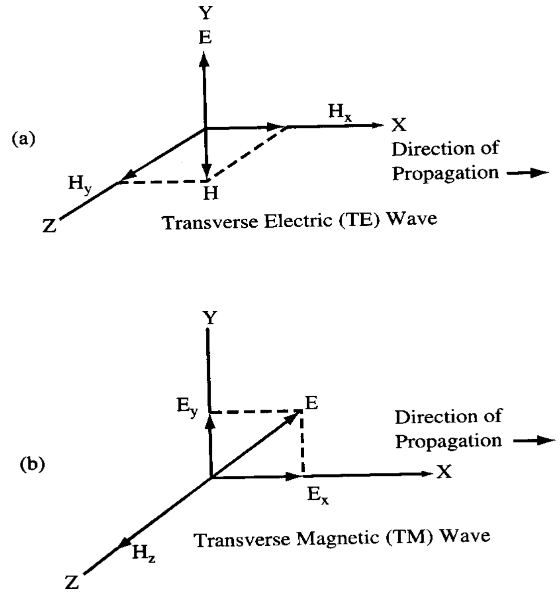
$$P_{\max} = \frac{\pi (0.035\times 2.54\times 0.001)^2 (3\times 10^6)^2}{377} \ln\frac{0.114}{0.035}$$
$$= 700W$$

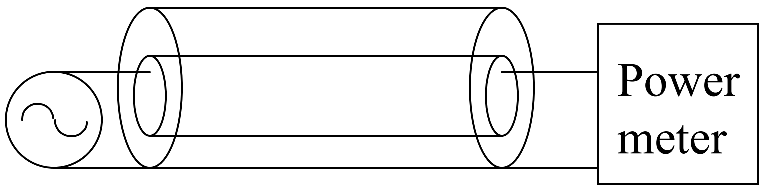
7. Attenuation

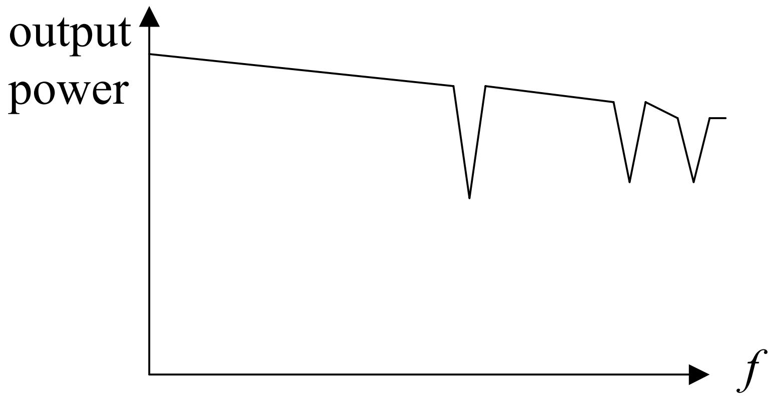
$$\alpha = \alpha_c + \alpha_d = \frac{R_s}{\eta \ln D/d}\left(\frac{1}{D} + \frac{1}{d}\right) + \frac{w\varepsilon''\eta}{2}, \eta = \sqrt{\frac{\mu}{\varepsilon'}}$$

8. Higher order modes can exist as $\lambda \approx$ line physical dimension









9. TE_{11} mode cutoff wavelength $\lambda_c \approx \pi(D+d)/2$, cutoff frequency

$$f_c = \frac{2c}{\pi(D+d)\sqrt{\varepsilon_r}}$$

→ the maximum operating frequency f_{max} of TEM mode is given as $0.95f_c$

→ the maximum allowable value of D to have only TEM mode is

$$D_{\max} = \frac{1.9c}{\pi f_{\max}(1+d/D)\sqrt{\varepsilon_r}} < \frac{0.95c}{\pi f_{\max}\sqrt{\varepsilon_r}}$$

→ reduce diameter to increase TEM mode operating frequency

→ lower the power capacity

→ optimize D/d

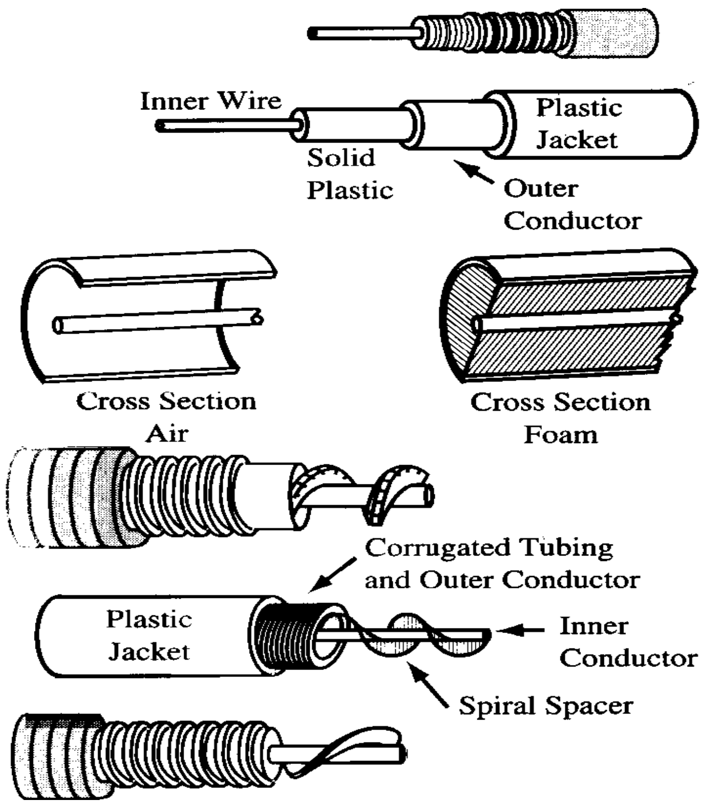
Ex. RG-142 coaxial cable, d=0.07in, D=0.232in, ε_r=2.2 → f_c=17GHz for TE_{11} mode → $f_{max} = 0.95f_c$ = 16GHz

10. Maximize P_{max} → optimum $D/d=\sqrt{e}$ → $Z_o=30\Omega$ (air-insulated line)
Minimize α_c → optimum $D/d=3.6$ → $Z_o=77\Omega$ (air-insulated line)
⇒ select $Z_o=50\Omega$ in the manufacture and use of connectors, measurement equipment, and standard components

11. Types of coaxial line

semi-rigid cable

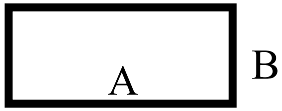
rigid cable



flexible cable

4.4 Rectangular waveguide

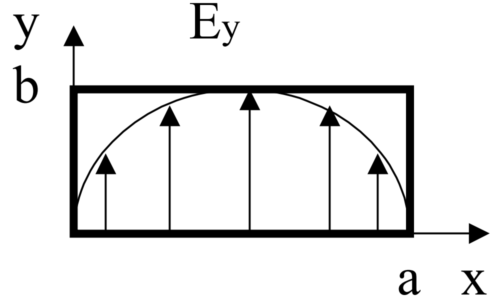
1. Structure: copper, brass or aluminum with inner side wall coated with gold or silver
2. Operating band/dimensions



EIA Designation	Minimum Frequency (GHz)	Size 1/100 in.	Inner Dimensions A (in.)	Inner Dimensions B (in.)
WR-2300	0.256	2300	23.000	11.500
WR-2100	0.281	2100	21.000	10.500
WR-1800	0.328	1800	18.000	9.000
WR-1500	0.328	1500	15.000	7.500
WR-1150	0.513	1150	11.500	5.750
WR-975	0.605	975	9.750	4.875
WR-770	0.766	770	7.700	3.850
WR-650	0.908	650	6.500	3.250
WR-510	1.158	510	5.100	2.550
WR-430	1.375	430	4.300	2.150
WR-340	2.737	340	3.400	1.700
WR-284	2.080	284	2.840	1.340
WR-229	2.579	229	2.290	1.145
WR-187	3.155	187	1.872	0.872
WR-159	3.714	159	1.590	0.795
WR-137	4.285	137	1.372	0.622
WR-112	5.260	112	1.122	0.497
WR-90	6.560	90	0.900	0.450
WR-75	7.873	75	0.750	0.375
WR-62	9.490	62	0.622	0.311
WR-51	11.578	51	0.510	0.255
WR-42	14.080	42	0.420	0.170
WR-34	17.368	34	0.340	0.170
WR-28	21.200	28	0.280	0.140
WR-22	26.350	22	0.224	0.112
WR-19	31.410	19	0.188	0.094

Frequency Band (GHz)	EIA	MIL or JAN	IEC	Great Britain
0.75–1.12	WR-975	RG-204/U	R-9	4
0.96–1.45	WR-770	RG-205/U	R-12	5
1.12–1.70	WR-650	RG-69/U	R-14	6
1.45–2.20	WR-510	RG-103/U	R-18	7
1.70–2.60	WR-430	RG-104/U	R-22	8
2.20–3.30	WR-340	RG-112/U	R-26	9a
2.60–3.95	WR-284	RG-48/U	R-32	10
3.30–4.90	WR-229	RG-340/U	R-40	11a
3.95–5.85	WR-187	RG-49/U	R-48	12
4.90–7.05	WR-159	RG-343/U	R-58	13
5.85–8.20	WR-137	RG-50/U	R-70	14
7.05–10.00	WR-112	RG-51/U	R-84	15
8.20–12.40	WR-90	RG-52/U	R-100	16
10.00–15.00	WR-75	RG-346/U	R-120	17
12.40–18.00	WR-62	RG-91/U	R-140	18
15.00–22.00	WR-51	none	R-180	19
18.00–26.50	WR-42	RG-53/U	R-220	20
22.00–33.00	WR-34	none	R-260	21
26.50–40.00	WR-28	RG-96/U	R-320	22
33.00–50.00	WR-22	RG-97/U	R-400	23
40.00–60.00	WR-19	RG-272/U	R-500	24
50.00–75.00	WR-15	RG-98/U	R-620	25

3. Dominant mode: TE_{10} mode ($k>k_c$)



$$\vec{E} = \frac{-jw\mu a}{\pi} A \sin\frac{\pi x}{a} e^{-j\beta z}\hat{y}$$

$$\vec{H} = \frac{j\beta a}{\pi} A \sin\frac{\pi x}{a} e^{-j\beta z}\hat{x} + A\cos\frac{\pi x}{a} e^{-j\beta z}\hat{z}$$

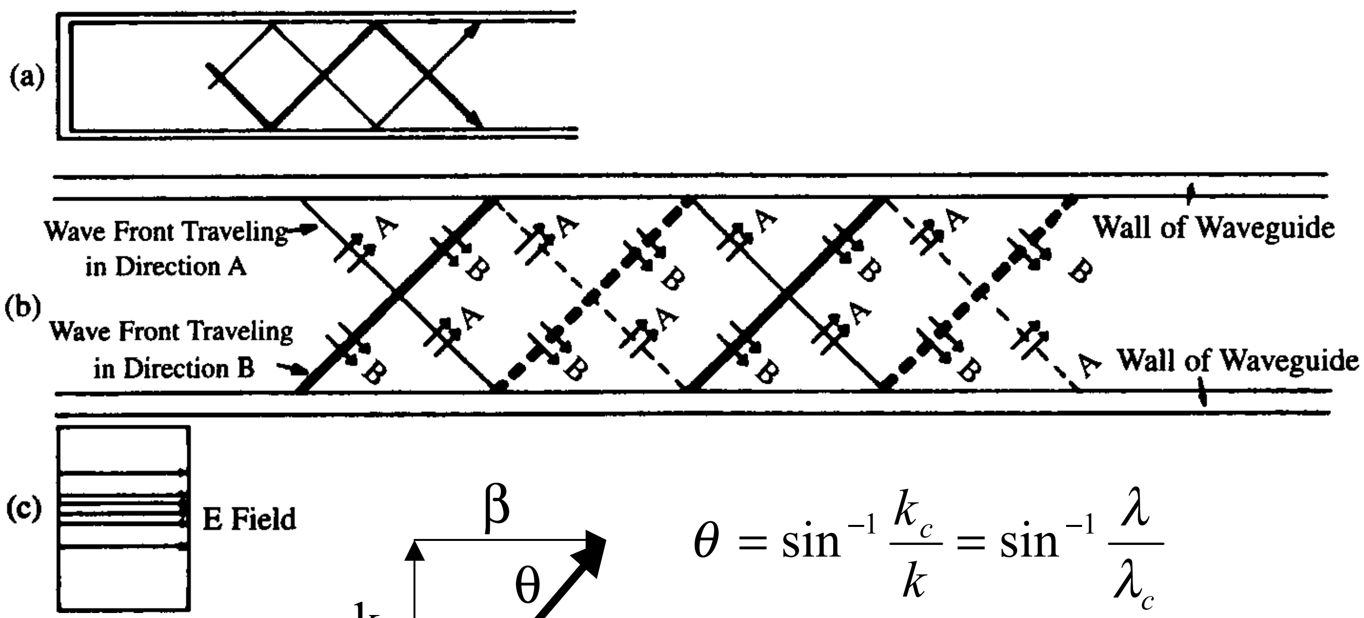
$$\beta = \sqrt{k^2 - k_c^2},\ k_c = \frac{\pi}{a} = \frac{2\pi}{\lambda_c},\ \lambda_c = 2a$$

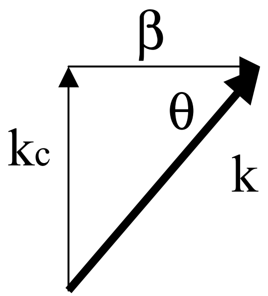
$$Z_{TE} = \frac{E_x}{H_y} = \frac{-E_y}{H_x} = \frac{w\mu}{\beta} = \frac{k\eta}{\beta} = \frac{377}{\sqrt{1-(\lambda/\lambda_c)^2}}$$

4. For TE_{10} mode, waveguide becomes a transverse resonator at its cutoff frequency
   i.e., $\beta = 0$ as $k = k_c$ or $\lambda_c/2 = a$, $f_c = v/\lambda_c$
   as $f < f_c$ $\beta = j\sqrt{k_c^2 - k^2}$: evanescent wave
   wave propagates as $f > f_c$ $\rightarrow$ waveguide is a high pass filter
5. Ex.4.3, 4.4 WR-90 waveguide with a=0.9in $\rightarrow$ λ_c=4.572cm, f_c=6.56GHz

6. TE_{10} mode propagates ≡ two plane waves obliquely propagate within the waveguide

$$E_y = \frac{-jw\mu a}{\pi} A \sin\frac{\pi x}{a} e^{-j\beta z} = \frac{-w\mu a}{2\pi} A(e^{jk_c x} - e^{-jk_c x})e^{-j\beta z}$$





$$\theta = \sin^{-1}\frac{k_c}{k} = \sin^{-1}\frac{\lambda}{\lambda_c}$$

$$f \uparrow \Rightarrow \theta \downarrow$$

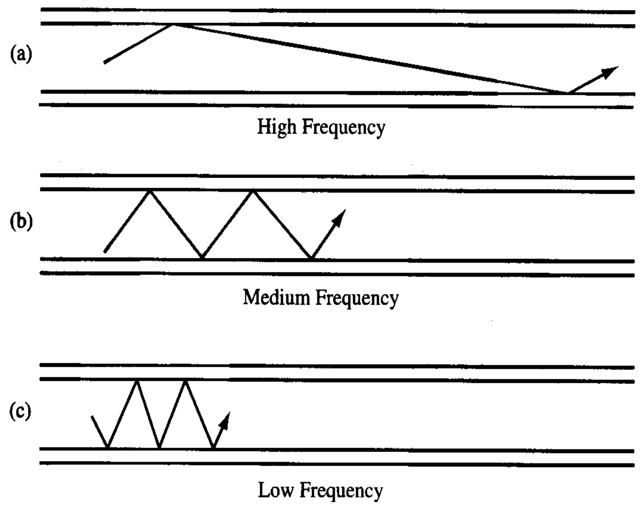
$$f = f_c \rightarrow \theta = 90^o$$

7. Ex.4.5 For WR-90 waveguide λ_c=4.572cm

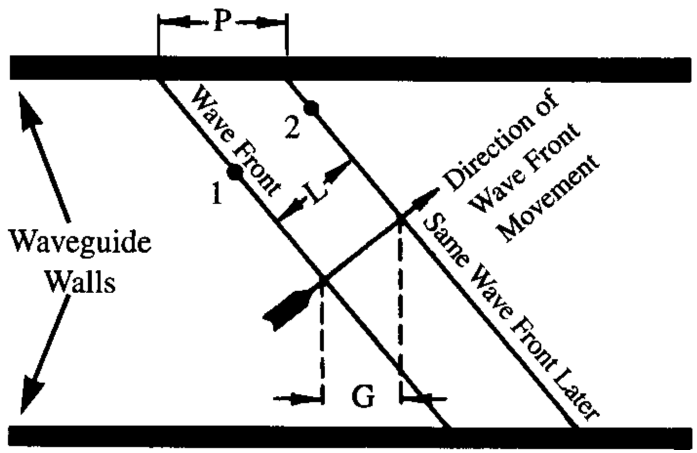
for 50GHz $\theta = \sin^{-1}\dfrac{0.6}{4.572} = 7.5^o$

for 10GHz $\theta = \sin^{-1}\dfrac{3}{4.572} = 41^o$

for 6.56GHz $\theta = \sin^{-1}\dfrac{4.572}{4.572} = 90^o$



8.



P $\rightarrow$ phase velocity v_p

G $\rightarrow$ group velocity v_g

L $\rightarrow$ velocity of light c

$v_p > c > v_g$

9. phase velocity $$v_p = \frac{w}{\beta} = \frac{c}{\sqrt{1-(\frac{\lambda}{\lambda_c})^2}}$$

group velocity $$v_g = c\sqrt{1-(\frac{\lambda}{\lambda_c})^2},\ v_p v_g = c^2$$

guide wavelength $$\lambda_g = \frac{2\pi}{\beta} = \frac{2\pi}{\sqrt{k^2 - k_c^2}} = \frac{\lambda}{\sqrt{1-(\lambda/\lambda_c)^2}} > \lambda = \frac{2\pi}{k}$$

cutoff wavelength for TE_{mn} or TM_{mn} mode $$\lambda_c = \frac{2}{\sqrt{(\frac{m}{a})^2 + (\frac{n}{b})^2}}$$

Ex. 4.6 WR-90 waveguide λ_g=3.6cm > λ=2.83cm at 10.6GHz

10. For each mode, it can be represented in terms of transmission line expression. For example, TE10 mode in a rectangular waveguide

$$\vec{E} = \frac{-jw\mu a}{\pi} A \sin\frac{\pi x}{a} e^{-j\beta z}\hat{y} : \text{propagate in + z - direction}$$

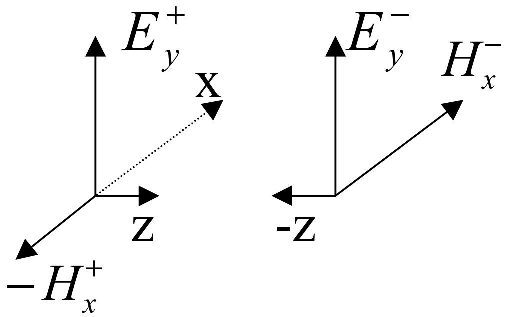
$$\Rightarrow E_y = (A^+ e^{-j\beta z} + A^- e^{j\beta z})\sin\frac{\pi x}{a} = (V^+ e^{-j\beta z} + V^- e^{j\beta z})\sin\frac{\pi x}{a} = V(z)\sin\frac{\pi x}{a}$$

$$\vec{H} = \frac{j\beta a}{\pi} A \sin\frac{\pi x}{a} e^{-j\beta z}\hat{x} + A\cos\frac{\pi x}{a} e^{-j\beta z}\hat{z} : \text{propagate in + z - direction}$$

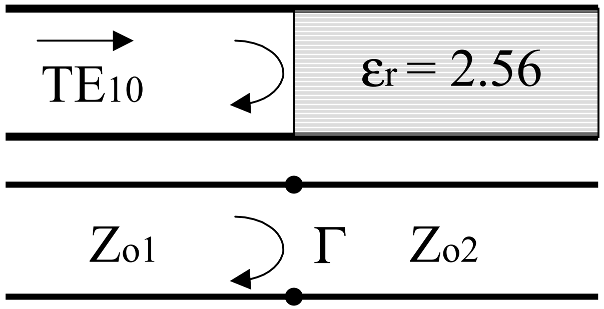
$$\Rightarrow H_x = \frac{-1}{Z_{TE}}(A^+ e^{-j\beta z} - A^- e^{j\beta z})\sin\frac{\pi x}{a} = \frac{-1}{Z_{TE}}(V^+ e^{-j\beta z} - V^- e^{j\beta z})\sin\frac{\pi x}{a}$$

$$= -(I^+ e^{-j\beta z} + I^- e^{j\beta z})\sin\frac{\pi x}{a} = -I(z)\sin\frac{\pi x}{a}$$

$$Z_{TE} = \frac{-E_y}{H_x} = \frac{w\mu}{\beta}$$



11. A WR-137 rectangular waveguide has a =3.485cm b=1.58cm, compute Γ at the dielectric interface for TE_{10} mode at 4.5GHz



TE_{10} exists in both media

Use transmission line approach

$$\beta = \sqrt{k^2 - k_c^2}, k_o = \frac{2\pi f}{c} = 94.25, k_c = \frac{\pi}{a} = 90.15$$

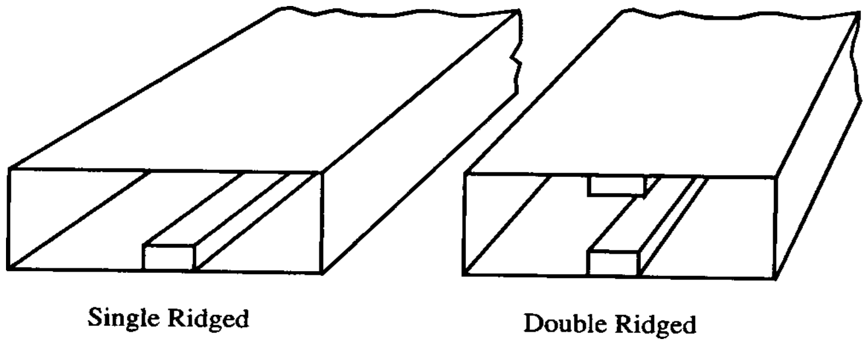
$$\Rightarrow f_{c1} = 4.3GHz, f_{c2} = 2.68GHz$$

$$\beta_1 = \sqrt{k_o^2 - k_c^2} = 27.5, \beta_2 = \sqrt{\varepsilon_r k_o^2 - k_c^2} = 120.89$$

$$Z_{o1} = \frac{k_o \eta_o}{\beta_1} = 1292, Z_{o2} = \frac{k_2 \eta_2}{\beta_2} = 294$$

$$\Rightarrow \Gamma = \frac{Z_{o2} - Z_{o1}}{Z_{o2} + Z_{o1}} = -0.629$$

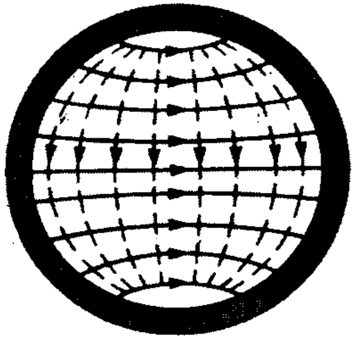
4.5 Ridged waveguide

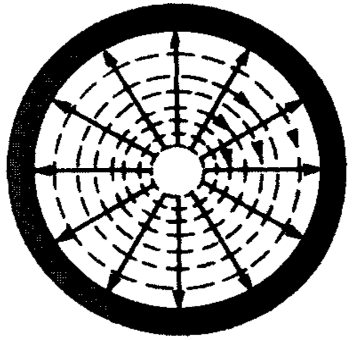


1. C increases $\rightarrow$ f_c decreases $\Rightarrow$ wider bandwidth
2. Tapered ridged waveguide can be used as a wideband impedance matching device. Ex. double ridged antenna

4.6 Circular waveguide

1. Dominant mode: TE11





Mode	$B_{(m,n)}$
TE_{01}	3.83
TE_{11} (Dominant)	1.84
TE_{21}	3.05
TE_{02}	7.02
TE_{12}	5.33
TE_{22}	6.71
TM_{01} (Symmetrical)	2.40
TM_{11}	3.83
TM_{21}	5.14
TM_{02}	5.52
TM_{12}	7.02
TM_{22}	8.42

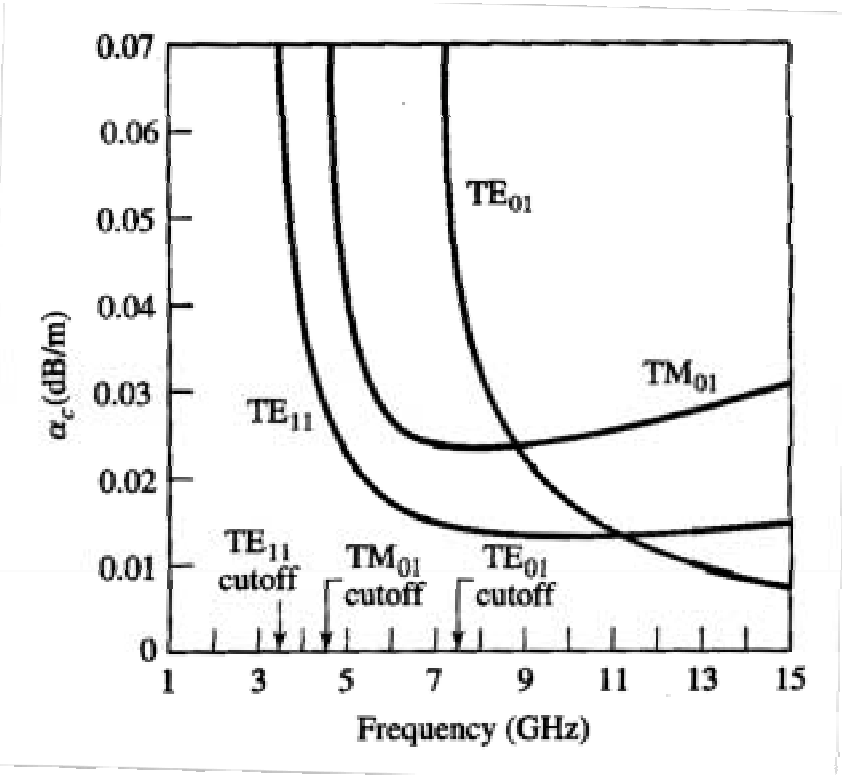
$$\vec{E} = \frac{-jw\mu}{k_c^2\rho} A\cos\phi J_1(k_c\rho)e^{-j\beta z}\hat{\rho} + \frac{jw\mu}{k_c} A\sin\phi J'_1(k_c\rho)e^{-j\beta z}\hat{\phi}$$

$$\vec{H} = \frac{-j\beta}{k_c} A\sin\phi J'_1(k_c\rho)e^{-j\beta z}\hat{\rho} + \frac{-j\beta}{k_c^2\rho} A\cos\phi J_1(k_c\rho)e^{-j\beta z}\hat{\phi}$$

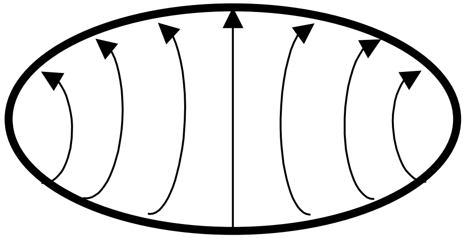
$$+ A\sin\phi J_1(k_c\rho)e^{-j\beta z}\hat{z}$$

$$B.C.\ E_\phi(\rho = a) = 0 \rightarrow J'_1(k_c a) = 0 \rightarrow \lambda_c = \frac{2\pi a}{B_{(1,1)}}$$

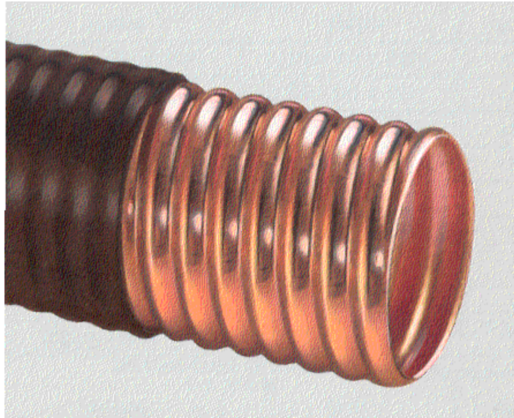
2. TE11: $\vec{E} = E_\rho \hat{\rho} + E_\phi \hat{\phi}$ can support v- and h-polarized waves, attenuation lower, power-capacity higher and operation bandwidth narrower.
3. Ex.4.7 a = 4cm TM_{02} mode λ_c = 4.55cm, f_c = 6.59GHz
4. Application in rotary joint for radar antenna



4.7 Elliptical waveguide



TE_{11} mode



1. Operating band/dimensions

Waveguide Operating Frequency, GHz	Standard Type No.	Standard Part No.	Premium Type No.	Premium Part No.
3.60-4.20	WE37	810250-001	WEP37	810250-002
4.40-5.00	WE44	810251-001	WEP44	810251-002
5.925-6.425	WE61	810265-001	WEP61	810265-002
6.425-7.125	WE65	810261-001	WEP65	810261-002
7.125-7.750	WE70	810254-001	WEP70	810254-002
7.125-8.50	WE71	810255-001	WEP71	810255-002
10.70-11.70	WE108	810264-001	WEP108	810264-002
11.70-13.20	WE130	810258-001	WEP130	810258-002
17.7-19.7	WE190	810260-001	WEP190	810260-002

MECHANICAL CHARACTERISTICS

Waveguide Type			WE61	WEP61	WE65	WEP65
Part No.: Standard, Premium			810265-001	810265-002	810261-001	810261-002
Weight, approx.	lbs./ft.	(kg/m)	.59	(.877)	.53	(.79)
Outer Diameters (Dimensions over Jacket—Major x Minor Axis)	in.	(mm)	2.19 × 1.27 (55.6 × 32.3)		2.03 × 1.18 (51.5 × 29.9)	
Bending Radius E-Plane H-Plane	in.	(mm)	12 (304.8) 30 (762.0)		12 (304.8) 24 (609.6)	
Max. Operating Pressure	psi	(bar)	10 (.7)		10 (.7)	
Max. Installation Length for 1 hoisting grip	ft.	(m)	330 (100)		330 (100)	
Standard Hanger Spacing	ft.	(m)	3 (.9)		3 (.9)	
Installation Temperature Range		°F (°C)	0 to +140 (−18 to +60)		0 to +140 (−18 to +60)	

ELECTRICAL CHARACTERISTICS

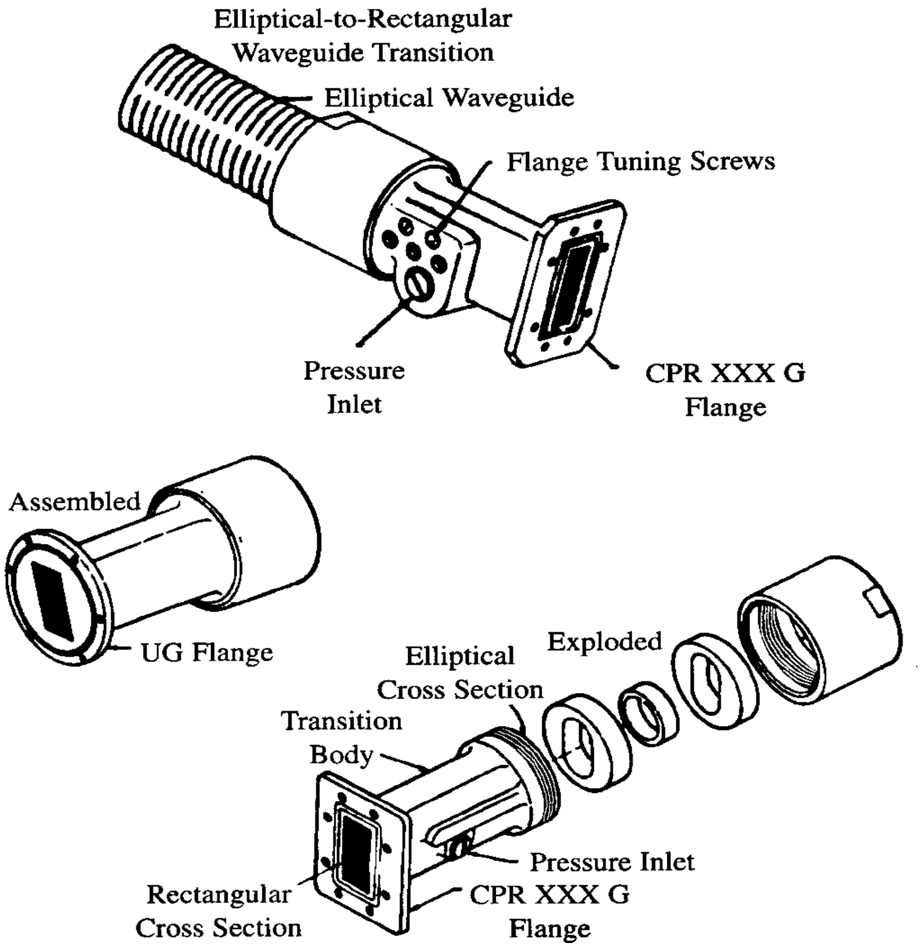
Waveguide Type		WE61	WEP61	WE65	WEP65
Waveguide Operating Frequency Range	GHz	5.925-6.425		6.425-7.125*	
Attenuation in dB/100 ft. (dB/100 m)		1.22 (4.00) @ 5.925 GHz 1.18 (3.87) @ 6.175 GHz 1.17 (3.83) @ 6.425 GHz		1.40 (4.59) @ 6.425 GHz 1.35 (4.43) @ 6.875 GHz 1.30 (4.26) @ 7.125 GHz	

* Available frequency range 5.925-7.125 GHz

ASSEMBLIES

Waveguide	Operating Frequency (GHz)	Connector Type No.	Connector Part No.	Flanges Mate To	VSWR Max. to 300 ft. (90 m) RMS	VSWR Max. to 300 ft. (90 m) Peak
WE61	5.925-6.425	C61-159ET C61-137ET	399268-101 399269-101	CPR159G CPR137G	1.07 1.07	1.15 1.15
WEP61	5.925-6.425	C61-159ET C61-137ET	399268-101 399269-101	CPR159G CPR137G	1.03 1.03	1.06 1.06
WE65	6.425-7.125*	C65-137E C65-137C	399208-107 399210-107	CPR137G UG-343/344U	1.07 1.07	1.15 1.15
WEP65	6.425-7.125*	C65-137ET C65-137CT	399208-105 399210-105	CPR137G UG-343/344U	1.03 1.03	1.06 1.06

2. Transition



3. They are most commonly used as feed lines for microwave system from antenna to equipment building.

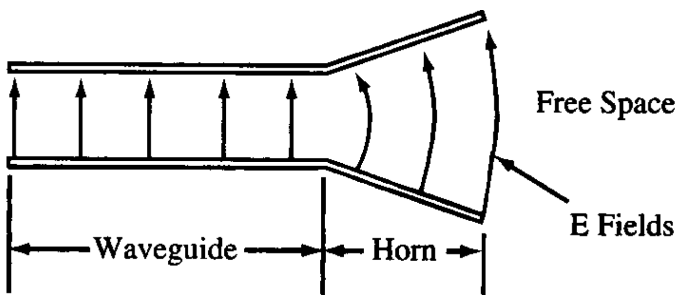
4.8 Waveguide discontinuities

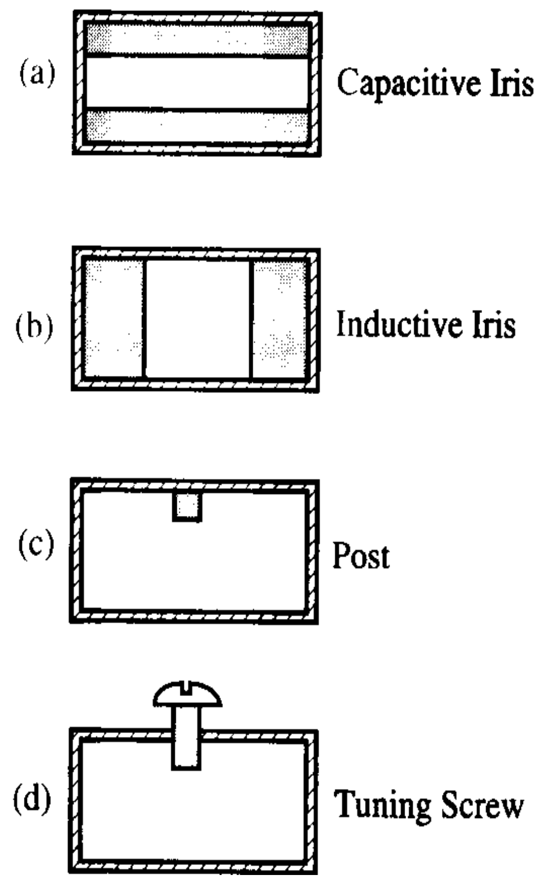
1. Impedance matching circuit
   Equivalent circuit components

   $\Delta E \Rightarrow C, \quad \Delta H \Rightarrow L$

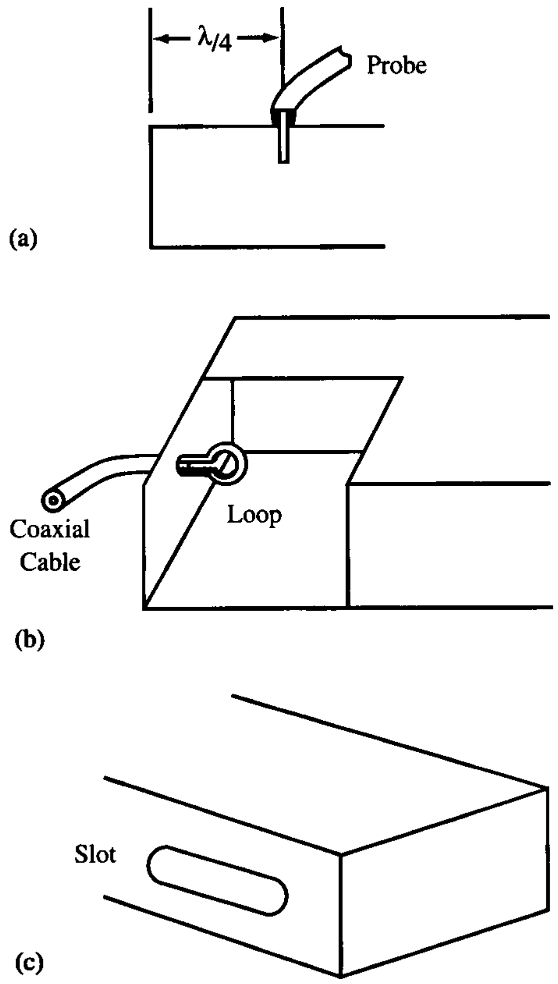
   constant E (V) $\Rightarrow$ parallel connection

   constant H (I) $\Rightarrow$ serial connection
2. Horn antenna





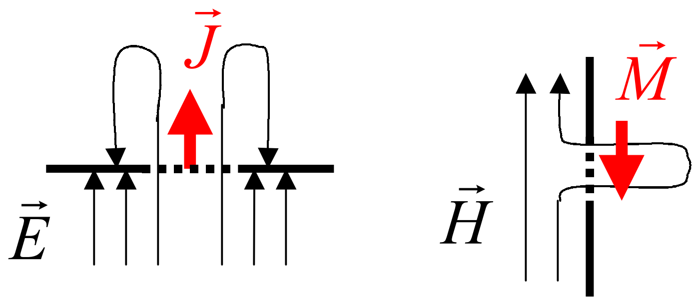
4.9 Methods of exciting waveguides



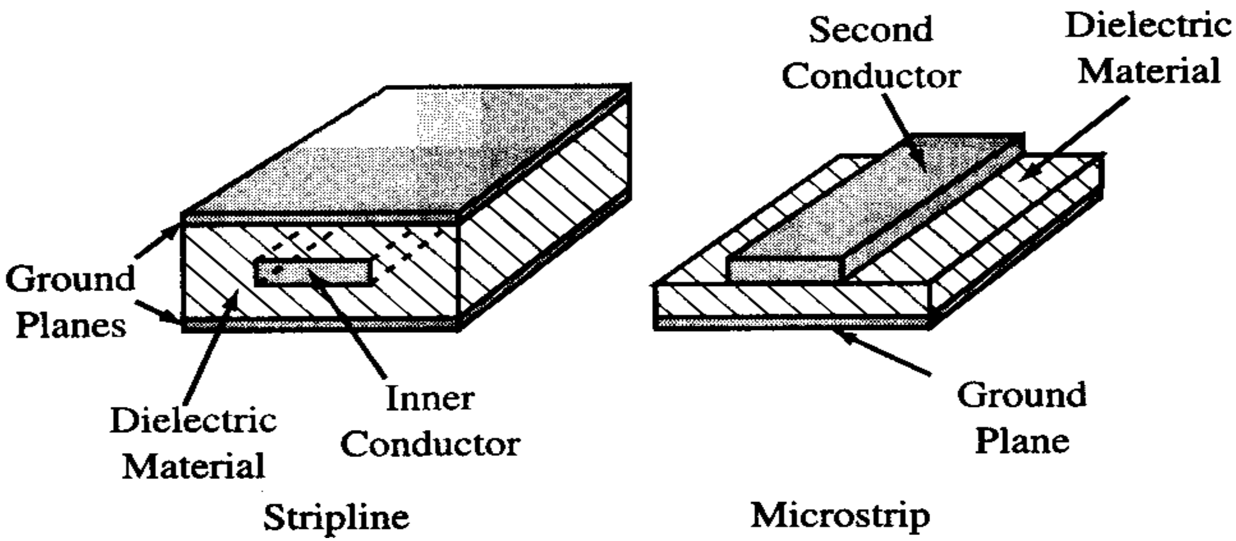
capacitive coupling $\longrightarrow \ \vec{E}$

inductive coupling $\longrightarrow \ \vec{H}$

slot (aperture) coupling $\rightarrow \vec{E}$ or $\vec{H}$



4.10 Stripline and microstrip



1. Stripline characteristics
   TEM mode operation as a “flattened” coaxial line
   wide operating band up to millimeter wave range
   fabrication by printed circuit
   components are not easy to access
   very low radiation loss

2. Microstrip characteristics

quasi-TEM mode operation

fabrication by printed circuit

devices can be bonded to strip (HMIC)

components are accessible

dc as well as ac signals can be transmitted

large variation in Zo

$\alpha_c > \alpha_d$

monolithic applications (MMIC)

structure is rugged and can withstand high voltages and power levels

power handling is best with BeO

used up to 300GHz or more

3. Microstrip equivalent circuit elements

series high impedance microstrip line	≡	series L
series low impedance microstrip line	≡	shunt C
shunt an open-circuit microstrip line	≡	shunt C
shunt a short-circuit microstrip line	≡	shunt L

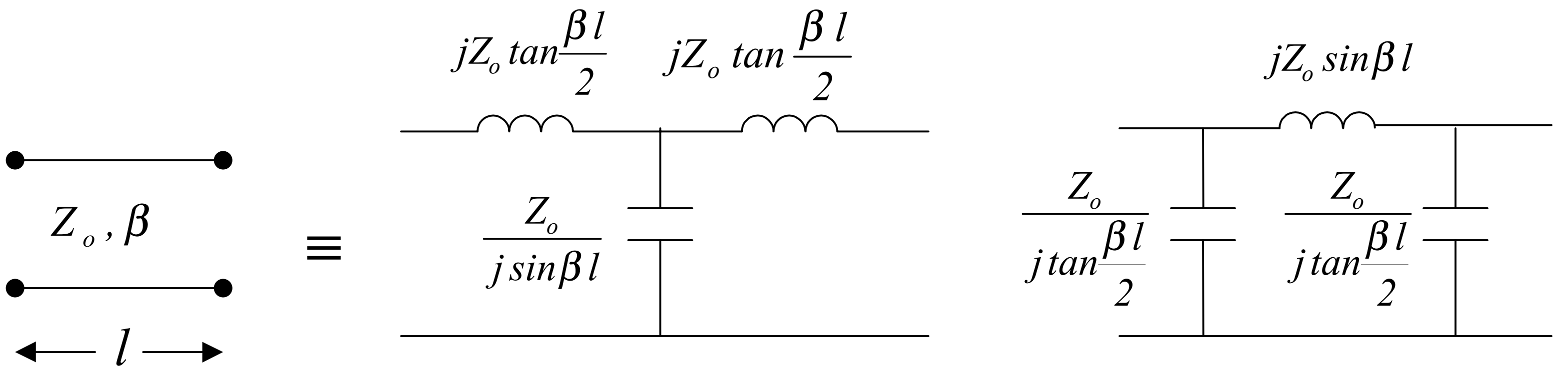
4. Open stub

$$\because Z_{in} = Z_o \frac{Z_L + jZ_o \tan \beta l}{Z_o + jZ_L \tan \beta l} \rightarrow Z_{in} = \frac{Z_o}{j \tan \beta l} \equiv \frac{1}{j\omega C}$$

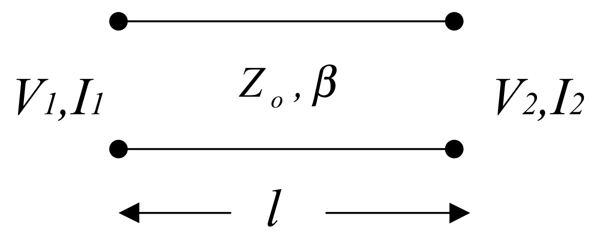
Short stub

$$Z_{in} = jZ_o \tan \beta l \equiv j\omega L$$

5. Transmission line



6. Derivation of T-equivalent circuit



$$[Z]=\begin{bmatrix} \dfrac{Z_o}{j\tan\beta l} & \dfrac{Z_o}{j\sin\beta l} \\ \dfrac{Z_o}{j\sin\beta l} & \dfrac{Z_o}{j\tan\beta l} \end{bmatrix}$$

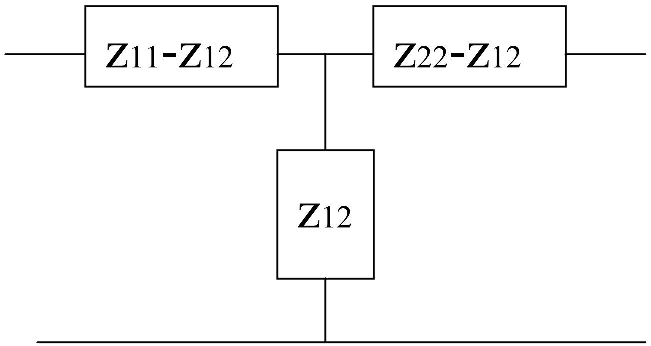
$$V(z)=V_o^+e^{-j\beta z}+V_o^-e^{j\beta z}\rightarrow V_1=V_o^++V_o^-(1),V_2=V_o^+e^{-j\beta l}+V_o^-e^{j\beta l}(2)$$

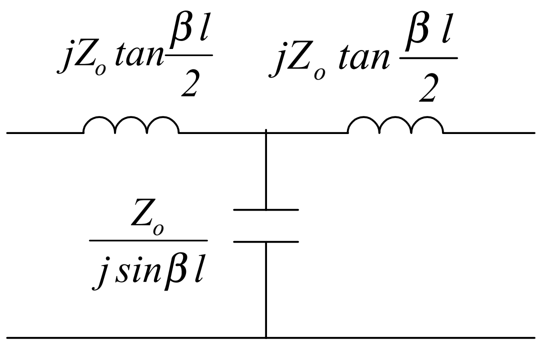
$$I(z)=\frac{V_o^+}{Z_o}e^{-j\beta z}-\frac{V_o^-}{Z_o}e^{j\beta z}\rightarrow I_1=\frac{V_o^+}{Z_o}-\frac{V_o^-}{Z_o}(3),I_2=\frac{V_o^+}{Z_o}e^{-j\beta l}-\frac{V_o^-}{Z_o}e^{j\beta l}(4)$$

$$(3),(4)\rightarrow V_o^+=\frac{1}{j2\sin\beta l}(I_1Z_oe^{j\beta l}-I_2Z_o),V_o^-=\frac{1}{j2\sin\beta l}(I_1Z_oe^{-j\beta l}-I_2Z_o)$$

$$\rightarrow V_1=\frac{Z_o}{j\tan\beta l}I_1-\frac{Z_o}{j\sin\beta l}I_2,V_2=\frac{Z_o}{j\sin\beta l}I_1-\frac{Z_o}{j\tan\beta l}I_2$$

open - circuit impedance matrix $\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}=\begin{bmatrix} z_{11} & z_{21} \\ z_{12} & z_{22} \end{bmatrix}\begin{bmatrix} I_1 \\ -I_2 \end{bmatrix}$



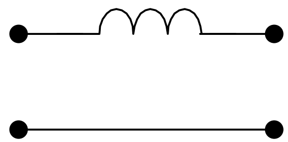


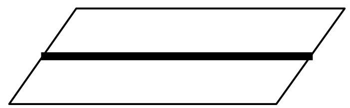
$$[Z]=\begin{bmatrix} \dfrac{Z_o}{j\tan\beta l} & \dfrac{Z_o}{j\sin\beta l} \\ \dfrac{Z_o}{j\sin\beta l} & \dfrac{Z_o}{j\tan\beta l} \end{bmatrix}$$

$$z_{11}-z_{12}=\frac{Z_o}{j\tan\beta l}-\frac{Z_o}{j\sin\beta l}=\frac{\cos\beta l-1}{j\sin\beta l}Z_o=\frac{-2\sin^2\frac{\beta l}{2}}{j2\sin\frac{\beta l}{2}\cos\frac{\beta l}{2}}Z_o=jZ_o\tan\frac{\beta l}{2}$$

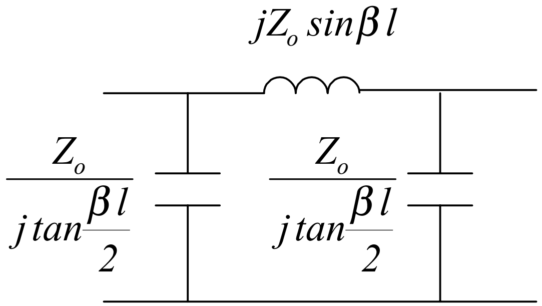
$$jwL \approx jZ_o\beta l$$

7. High impedance line $\approx$

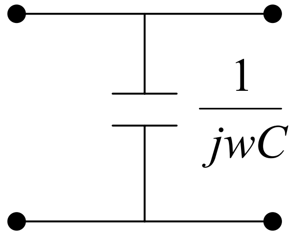




8. Low impedance line

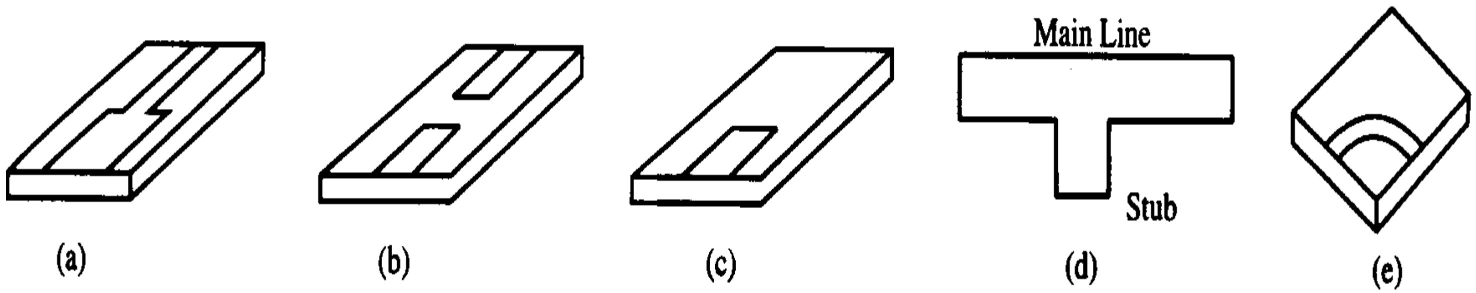


$\approx$



$$\frac{1}{jwC} \approx \frac{Z_o}{j\beta l}$$

9. Microstrip discontinuities



<u>Homework #4</u> (due 2 weeks)
Chap.4: problems 1-12