Chapter 2 Transmission line fundamentals

2.1 Introduction
    comparison of microwave transmission lines, losses,
    skin depth, ac resistance
2.2 Concepts of two-wire lines
    transmission line equation, solution, characteristic impedance,
    input impedance
2.3 Free space characteristic impedance
    intrinsic impedance, wave impedance
2.4 Matched terminations
    load match, source match, conjugated match
    maximum power transfer
2.5 Propagation velocity, velocity factor, and attenuation constant
2.6 Standing waves
    standing wave solution
2.7 Reflection coefficient and return loss

2.8 Voltage standing-wave radio

2.9 Open and shorted terminators

effects of termination, equivalent circuits

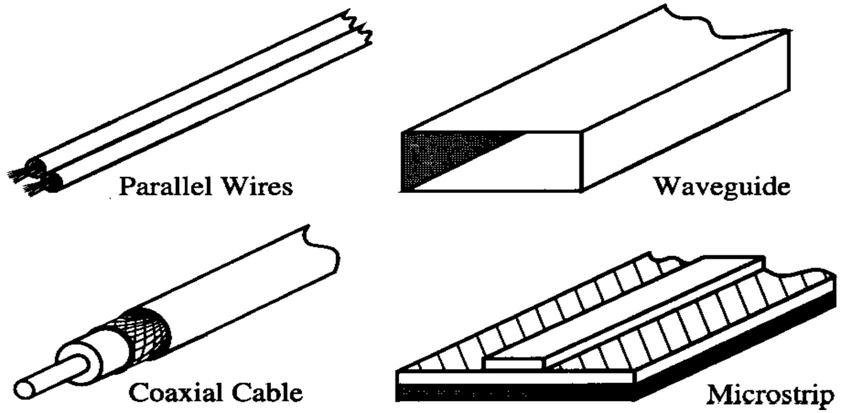
2.10 Transmission line sections

$\lambda/4$ and $\lambda/2$ lines

2.11 Time-domain reflectometry

operation principle

2.1 Introduction



1.

Characteristics	coaxial line	waveguide	microstrip
preferred mode	TEM	TE_{10}	quasi-TEM
dispersion	none	medium	low
bandwidth	high	low	high
loss	medium	low	high
power capacity	medium	high	low
physical size	large	large	small
easy of fabrication	medium	medium	easy
integration with other components	hard	hard	easy

2. Transmission line characteristics: Z_o and $\gamma=\alpha+j\beta$,
$\alpha = \alpha_c$(conductor loss) + α_d (dielectric loss) + radiation loss

3. Conductor loss depends on the metal conductivity and surface area

$$P_{lc} = \frac{1}{2} R_s \int_S \left|\vec{J}_s\right|^2 ds = \frac{1}{2} R_s \int_S \left|\vec{H}_t\right|^2 ds, \quad R_s = \frac{1}{\sigma\delta_s} = \sqrt{\frac{w\mu}{2\sigma}}$$

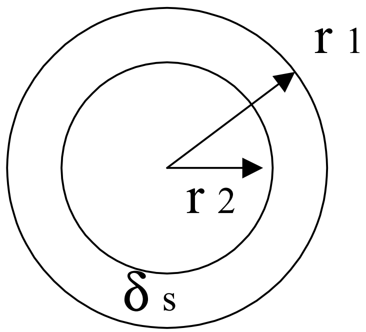
4. Dielectric loss depends on the dielectric loss tangent and volume

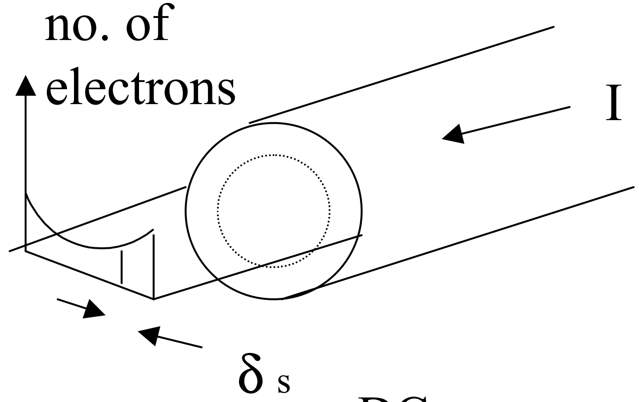
$$P_{ld} = \frac{1}{2} w\varepsilon'' \int_V \left|\vec{E}\right|^2 dv, \quad \varepsilon = \varepsilon' \text{-} j\varepsilon'', \quad \tan\delta = \frac{\varepsilon''}{\varepsilon'}$$

5. Skin depth $\delta_s = \frac{1}{\alpha} = \sqrt{\frac{2}{w\mu\sigma}}$

copper $\sigma = 5.813 \times 10^7 S/m \rightarrow \delta_s = 0.66um$ at 10GHz

6. AC resistance





$$\text{AC resistance} = \frac{\text{DC area}}{\text{AC area}} \times \text{DC resistance}$$

$$\Rightarrow \text{f}\uparrow, \delta_s \downarrow, \ \text{AC reistance}\uparrow, \ \alpha\uparrow$$

Ex. No.22 wire with diameter 0.644mm, DC resistance 60.97Ω/km, δ_s=21.6um at 10MHz

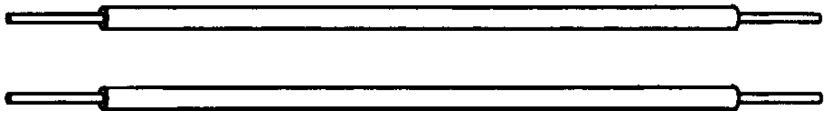
$$\text{DC area} = \pi\ \text{r}_1^2 = \pi \times 0.322^2 = 0.3257\,mm^2$$

$$\text{AC area} = 0.3257 - \pi(0.322 - 0.0216)^2 = 0.0422\,mm^2$$

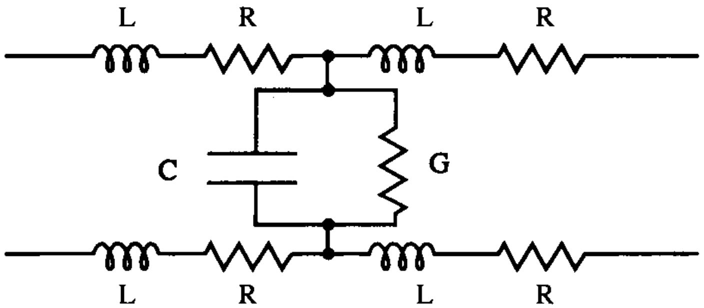
$$\text{AC resistance} = \frac{\text{DC area}}{\text{AC area}} \times \text{DC resistance} = 450.6\,\Omega/\text{km}$$

2.2 Concept of two-wire lines

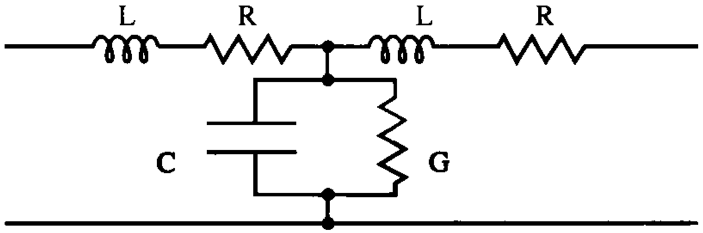
two-wire lines



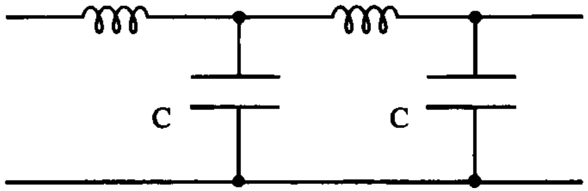
equivalent transmission line



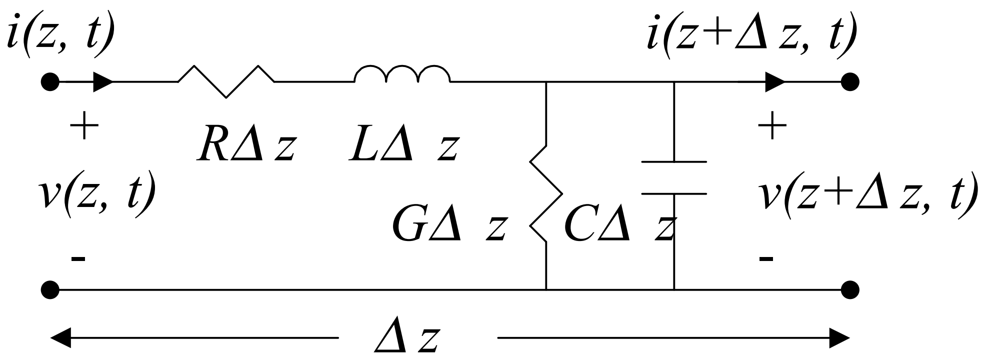
simplified line



lossless line



1.



distributed constants

R, L: conductor resistance, inductance/unit length (series elements)

G, C: dielectric conductance, capacitance/unit length (parallel elements)

KVL, KCL ⇨ time-domain transmission line, or telegrapher equation

$$\frac{\partial v(z,t)}{\partial z} = -Ri(z,t) - L\frac{\partial i(z,t)}{\partial t} \qquad \Rightarrow \qquad \frac{dV(z)}{dz} = -(R + jwL)I(z)$$

$$\frac{\partial i(z,t)}{\partial z} = -Gi(z,t) - C\frac{\partial v(z,t)}{\partial t} \qquad \qquad \frac{dI(z)}{dz} = -(G + jwC)V(z)$$

time-harmonic form

$$\Rightarrow \quad \frac{d^2V(z)}{dz^2} - \gamma^2 V(z) = 0, \qquad \frac{d^2I(z)}{dz^2} - \gamma^2 I(z) = 0 \quad \text{wave equation}$$

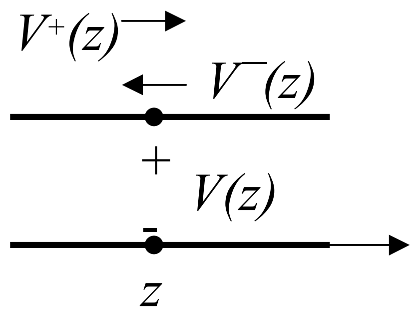
$$\gamma \equiv \sqrt{(R + jwL)(G + jwC)} \equiv \alpha + j\beta \quad \text{propagation constant}$$

2. Travelling wave solution

$$V(z) = V^{+}(z) + V^{-}(z) = V_o^{+} e^{-\gamma z} + V_o^{-} e^{\gamma z}$$

$$I(z) = I^{+}(z) + I^{-}(z) = I_o^{+} e^{-\gamma z} + I_o^{-} e^{\gamma z} = \frac{V_o^{+}}{Z_o} e^{-\gamma z} - \frac{V_o^{-}}{Z_o} e^{\gamma z}$$

$$\Rightarrow \; Z_o \equiv \sqrt{\frac{R + jwL}{G + jwC}} = \frac{V_o^{+}}{I_o^{+}} = -\frac{V_o^{-}}{I_o^{-}}$$



time-domain solution

$$v(z,t) = \left|V_o^{+}\right| e^{-\alpha z} \cos(wt - \beta z + \angle V_o^{+}) + \left|V_o^{-}\right| e^{\alpha z} \cos(wt + \beta z + \angle V_o^{-})$$

$$i(z,t) = \left|I_o^{+}\right| e^{-\alpha z} \cos(wt - \beta z + \angle I_o^{+}) + \left|I_o^{-}\right| e^{\alpha z} \cos(wt + \beta z + \angle I_o^{-})$$

3. characteristic impedance

$$Z_0 \equiv \frac{V_0^+}{I_0^+}\left(= \frac{V^+(z)}{I^+(z)}\right) = -\frac{V_0^-}{I_0^-}\left(= -\frac{V^-(z)}{I^-(z)}\right)$$

input impedance

$$Z_{in}(z) \equiv \frac{V(z)}{I(z)}$$

4. As a transmission line is terminated with Z_o, $Z_{in}(z)=Z_o$.
5. Ex.2.1 RG59 cable L=370nH/m, C=67pF/m $\rightarrow$

$$Z_o = \sqrt{\frac{L}{C}} = 74.3\Omega$$

2.3 Free space characteristic impedance

1. characteristic impedance of the medium (intrinsic impedance) $\eta = \sqrt{\dfrac{\mu}{\varepsilon}}$

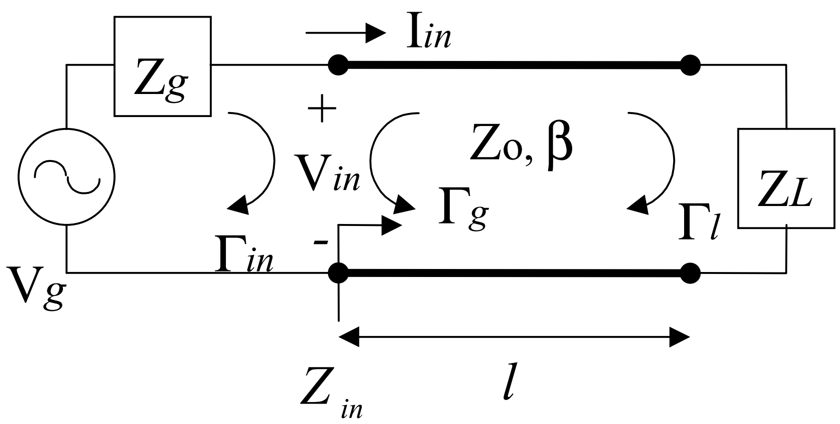
   wave impedance of the particular mode of wave $Z_w = \dfrac{E_t}{H_t}$

   characteristic impedance of transmission line $Z_o = \dfrac{V_o^+}{I_o^+}$

   input impedance at a port of circuit $Z_{in}(z) = \dfrac{V(z)}{I(z)}$

2. For free space $\mu_o = 4\pi \times 10^{-7} H/m, \varepsilon_o = 8.85 \times 10^{-12} F/m$
   $Z_o = 120\pi\Omega = 377\Omega$
3. Antenna is an impedance transformer for a maximum power transfer to/from free space

2.4 Matched terminations



$$P_{in} = \frac{1}{2}\mathrm{Re}(V_{in} I_{in}^*)$$

$$= \frac{1}{2}\mathrm{Re}\left[\frac{V_g}{Z_g + Z_{in}} Z_{in} \left(\frac{V_g}{Z_g + Z_{in}}\right)^*\right]$$

$$= \frac{1}{2}|V_g|^2 \frac{R_{in}}{(R_{in} + R_g)^2 + (X_{in} + X_g)^2}$$

1. Load match: $Z_L = Z_o$ no reflected wave $\rightarrow Z_{in}(z) = Z_o$
   $\equiv$ infinite long line with Z_o

$$P_{in} = \frac{1}{2}|V_g|^2 \frac{Z_o}{(Z_o + R_g)^2 + X_g{}^2}$$

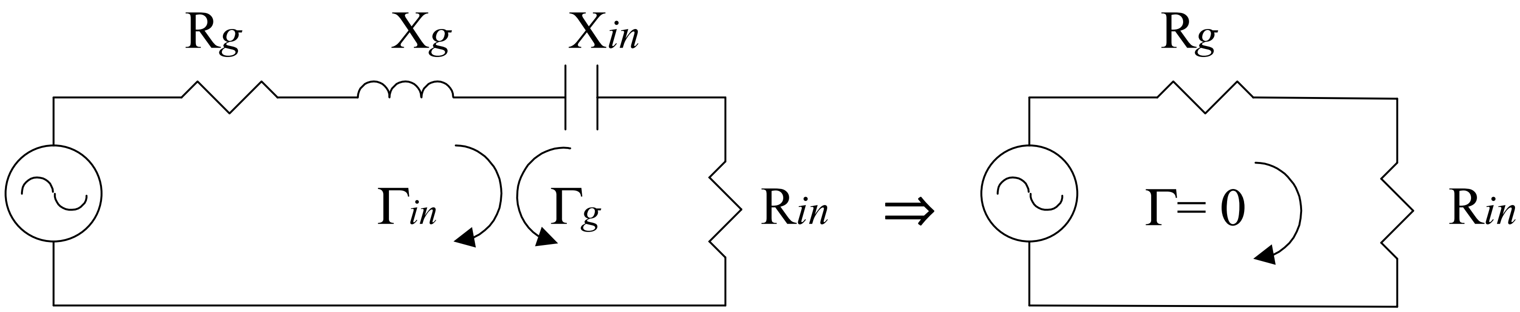
source match: $Z_{in} = Z_g \; (\Gamma_{in} = 0)$

$$P_{in} = \frac{1}{2}|V_g|^2 \frac{R_g}{4(R_g + X_g)^2}$$

2. Conjugate match

$$Z_{in} = Z_g^{\,*} \;(\Gamma_g \neq 0, \Gamma_{in} \neq 0) \Leftarrow \text{maximum power transfer}$$

$$P_{in,max} = \frac{1}{2}\left|V_g\right|^2 \frac{1}{4R_g} \qquad \frac{\partial P_{in}}{\partial R_{in}} = 0, \; \frac{\partial P_{in}}{\partial X_{in}} = 0$$



3. In practice, using matching circuits and lossless lines to have $Z_g = Z_L = Z_{in} = Z_o$ (matched terminations) for maximum power transferred from source to the load.

2.5 Propagation velocity, velocity factor, and attenuation constant

1. phase velocity $v_p = \frac{w}{\beta}$ group velocity $v_g = (\frac{d\beta}{dw})^{-1}$

   TEM transmission line $\beta = w\sqrt{\mu\varepsilon} = w\sqrt{LC}$

   propagation velocity $v = \frac{w}{\beta} = \frac{1}{\sqrt{LC}} = \frac{c}{\sqrt{\varepsilon_r}} = cv_f$

   velocity factor $v_f = \frac{1}{\sqrt{\varepsilon_r}}$

2. Ex. 2.2, 2.3 RG-59 cable $\mu_o = 4\pi\times10^{-7}H/m, \varepsilon_o = 8.85\times10^{-12}F/m$

   v= 0.67c, ε_r =2.25

3. Physical length of transmission line

   = physical length in free-space wavelength $\times$ vf

4. Low-loss line, R<<wL, G<<wC

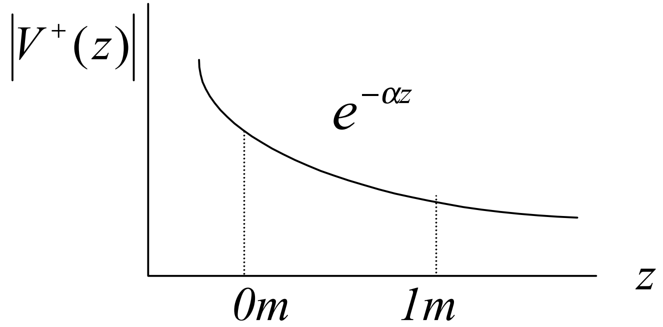
   $$\alpha \approx \frac{1}{2}(\frac{R}{Z_o} + GZ_o), \beta \approx w\sqrt{LC}, Z_o \approx \sqrt{\frac{L}{C}}$$

5. attenuation constant

$$|V^{+}(1m)| = |V^{+}(0m)|e^{-\alpha}$$

$$\alpha = \ln\frac{|V^{+}(0m)|}{|V^{+}(1m)|} \quad \text{(Neper)}$$

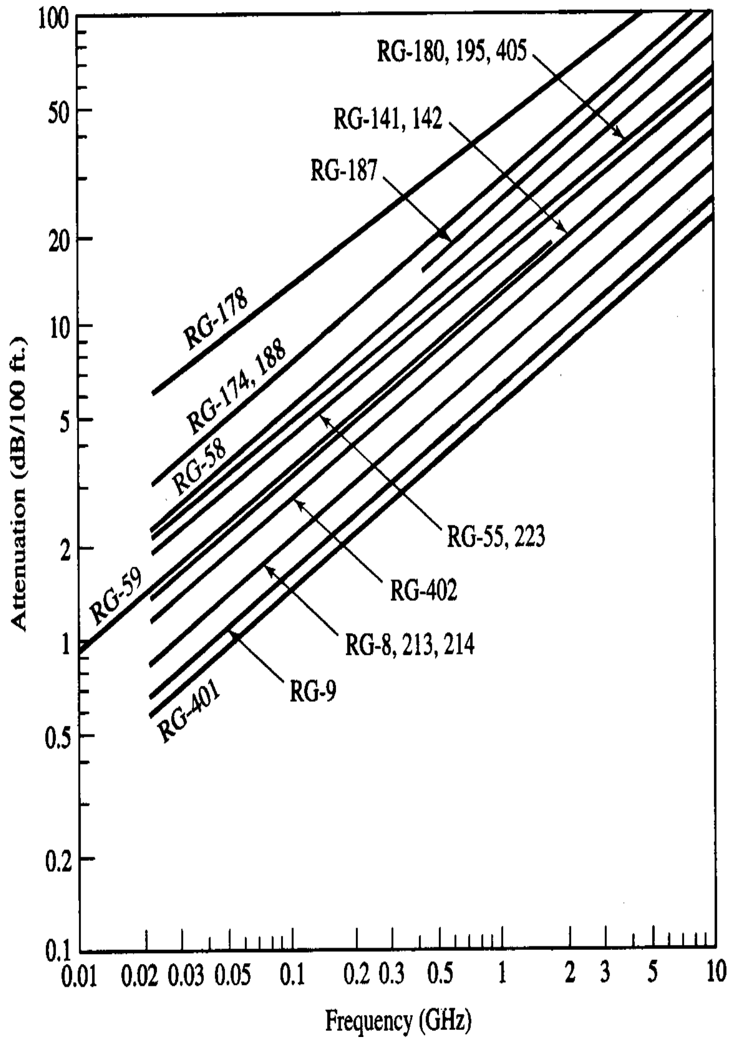
$$= 20\log\frac{|V^{+}(0m)|}{|V^{+}(1m)|} \text{(dB)}, \quad 1\text{Np} = 8.68\text{dB}$$



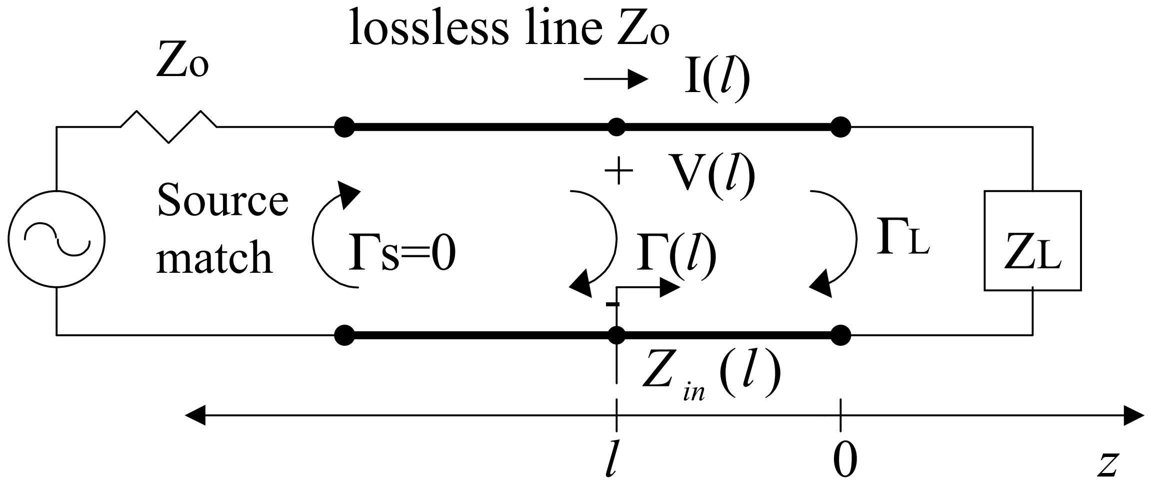
JAN Type No.	Outside Diameter (mm)	Maximum Operating Frequency (MHz)	Attenuation (dB/100m) @ 450 MHz	Attenuation (dB/100m) @ 1500 MHz	Attenuation (dB/100m) @ 6000 MHz	Velocity Factor
RG-213/U	10.29	1000	16.4			0.659
RG-213/U Commercial	10.29	–	9.2	19.0	48.2	0.84
RG-214/U	10.79	11,000	18.0	37.1	93.8	0.659
RG-142B/U	4.95	12,400	27.6	55.4	135	0.695
RG-393/U	9.91	11,000	16.1	32.8	79.4	0.695
RG-6/U	8.43	300	22.0	45.9	–	0.659
RG-11/U	10.29	1000	16.7	–	–	0.659

Cable Type	Impedance (Ω)	Dielectric Material†	Overall Diameter (In.)	Dielectric Diameter (In.)	Maximum Operating Voltage
RG–8A/U	52	P	0.405	0.285	5000
RG–9B/U	50	P	0.425	0.285	5000
RG–55/U	54	P	0.216	0.116	1900
RG–58/U	50	P	0.195	0.116	1900
RG–59/U	75	P	0.242	0.146	2300
RG–141/U	50	T	0.190	0.116	1900
RG–142/U	50	T	0.206	0.116	1900
RG–174/U	50	—	0.100	0.060	1500
RG–178/U	50	T	0.075	0.036	750
RG–180/U	95	T	0.137	0.103	750
RG–187/U	75	T	0.110	0.060	1200
RG–188/U	50	—	0.110	0.060	—
RG–195/U	95	T	0.155	0.102	1500
RG–213/U	50	P	0.405	0.285	5000
RG–214/U	50	P	0.425	0.285	5000
RG–223/U	50	P	0.216	0.116	1900
RG–401	50	T	0.250	0.208	—
RG–402	50	T	0.141	0.118	—
RG–405	50	T	0.087	0.066	—

† P: Polyethylene, T: Teflon



2.6 Standing waves



1. Standing wave solution: incident wave + reflected wave

$$V(z) = V_o^+ e^{-j\beta z} + V_o^- e^{j\beta z}$$

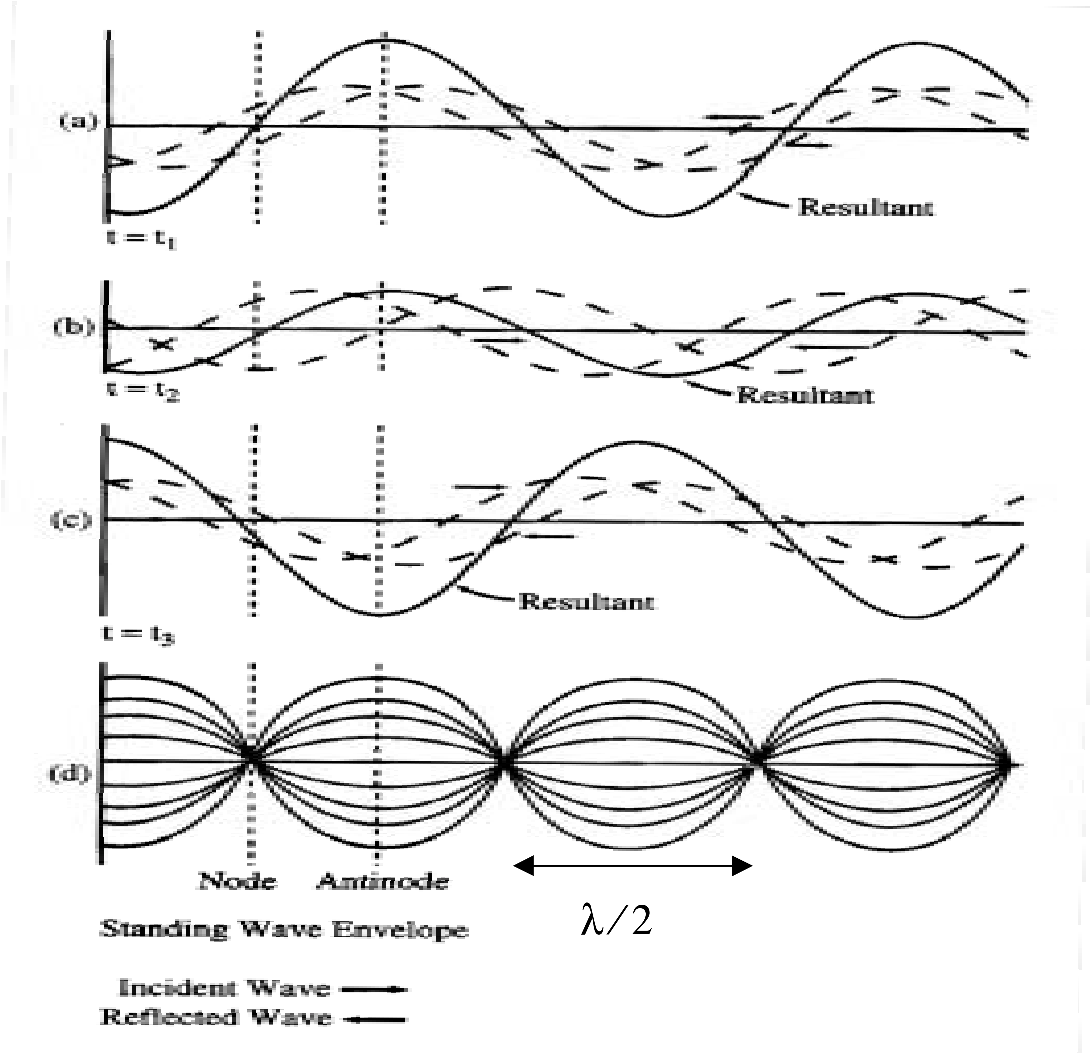
$$I(z) = I_o^+ e^{-j\beta z} + I_o^- e^{j\beta z} = \frac{V_o^+}{Z_o} e^{-j\beta z} - \frac{V_o^-}{Z_o} e^{j\beta z}$$

Time-average power flow

$$P_{av}(z) \equiv \frac{1}{T}\int_0^T v(z,t)i(z,t)dt = \frac{1}{2}\mathrm{Re}\left[V(z)I^*(z)\right] = \frac{1}{2}\frac{\left|V_o^+\right|^2}{Z_o}\left(1-\left|\Gamma_L\right|^2\right)$$

constant incident wave reflected wave

2. Standing wave pattern



2.7 Reflection coefficient and return loss

1. Reflection coefficient

$$\Gamma(l) \equiv \frac{V^-(l)}{V^+(l)} = \frac{V_o^- e^{-j\beta l}}{V_o^+ e^{j\beta l}} = \Gamma_L e^{-j2\beta l} = e^{-j\beta l}\Gamma_L e^{-j\beta l}$$

$0 \le |\Gamma| \le 1$ for a passive load

$|\Gamma| > 1$ for an active load

2. $\Gamma = |\Gamma| \angle\Gamma = \rho \angle \Gamma$

$|\Gamma(l)| = |\Gamma_L|$ remains the same anywhere along a lossless line.

3. Input impedance $Z_{in}(l) \equiv \dfrac{V(l)}{I(l)} = Z_o \dfrac{Z_L + jZ_o \tan \beta l}{Z_o + jZ_L \tan \beta l}$

4. $Z_{in} \longleftrightarrow \Gamma$

$$Z_{in}(z) = Z_o \frac{1+\Gamma(z)}{1-\Gamma(z)}, \qquad \Gamma(z) = \frac{Z_{in}(z) - Z_o}{Z_{in}(z) + Z_o}$$

$|\Gamma| > 1 \rightarrow \mathrm{Re}\{Z_{in}\} < 0$, negative resistance for an active load

5. Ex. 2.4 R_L=100Ω, R_o=50Ω → Γ =1/3

Ex. 2.5 R_L=25Ω, R_o=50Ω → Γ = -1/3 = 1/3 ∠180°

Ex. 2.6 Z_L=40-j2025Ω, Z_o=50Ω → Γ = 0.24 ∠-103.6°

6. Power reflection coefficient ρ^2, transmitted power $1-\rho^2$

7.

$$RL \equiv -20\log|\Gamma_L| \quad (dB) = -20\log\frac{V^-}{V^+} = -10\log\frac{P^-}{P^+}$$

$$\begin{array}{llll} e.g., \quad \Gamma_L = & 1 & 0.1 & 0 \\ \quad RL = & 0dB & 20dB & \infty dB \\ \quad \text{VSWR} & \infty & 1.22 & 1 \end{array}$$

all incident power reflected — matched load

worst load "no return loss" — "∞ return loss"

8. Ex. 2.7 ρ =1/3 → power reflection coefficient 1/9

Ex. 2.8 incident power 30mW, reflected power 1uW

→ RL = -10log(1/30000) = 44.8dB

Ex. 2.9 ρ = 0.2 → RL= -20log(0.2) =14dB

2.8 Voltage standing-wave ratio

1. Voltage standing-wave ratio, VSWR

$$VSWR \equiv \frac{V_{max}}{V_{min}} = \frac{1+|\Gamma_L|}{1-|\Gamma_L|}$$

$$|\Gamma_L| = \frac{VSWR - 1}{VSWR + 1}$$

$$1(matched) \le VSWR \le \infty(|\Gamma_L| = 1)$$

2. Ex. 2.11 ρ = 0.2 → VSWR = 1.5
   Ex. 2.12 VSWR= 3 → ρ = 0.5
3. VSWR = Rmax/Zo= Zo/Rmin

$$R_{max} = \frac{V_{max}}{I_{min}} = \frac{|V^+|+|V^-|}{|I^+|-|I^-|} = \frac{|V^+|+|V^-|}{(|V^+|-|V^-|)/Z_o}$$

$$= VSWR \times Z_o$$

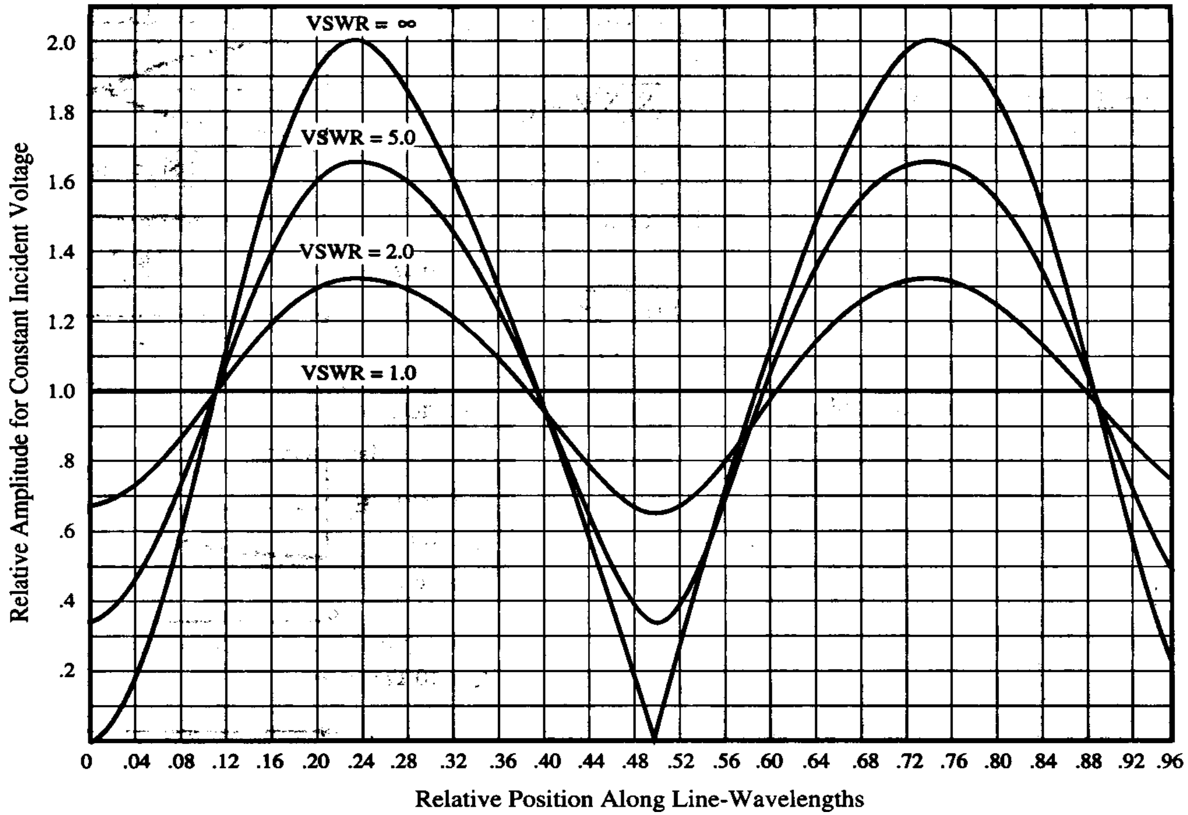
4. Ex. 2.14 VSWR= 4, Zo = 50Ω → Zmax= 200 Ω, Zmin= 12.5 Ω

VSWR	Return Loss	Reflection Coefficient (ρ)
1.01	46.06	0.0050
1.02	40.08	0.0099
1.03	36.60	0.0148
1.04	34.15	0.0196
1.05	32.25	0.0244
1.06	30.71	0.0291
1.07	29.41	0.0338
1.08	28.29	0.0385
1.09	27.31	0.0431
1.10	26.44	0.0476
1.11	25.65	0.0521
1.12	24.94	0.0566
1.13	24.28	0.0611
1.14	23.68	0.0654
1.15	23.12	0.0698
1.20	20.82	0.0909
1.25	19.08	0.1111
1.30	17.69	0.1304
1.40	15.56	0.166
1.50	13.97	0.20
2.0	9.54	0.33
3.0	6.021	0.50
4.0	4.437	0.6
5.0	3.522	0.666
10.0	1.743	0.818
20.0	0.869	0.904
30.0	0.579	0.935
40.0	0.434	0.9512
50.0	0.347	0.960
60.0	0.290	0.967
70.0	0.248	0.971
80.0	0.217	0.975
90.0	0.193	0.978
100.0	0.174	0.980

5. Impedance match

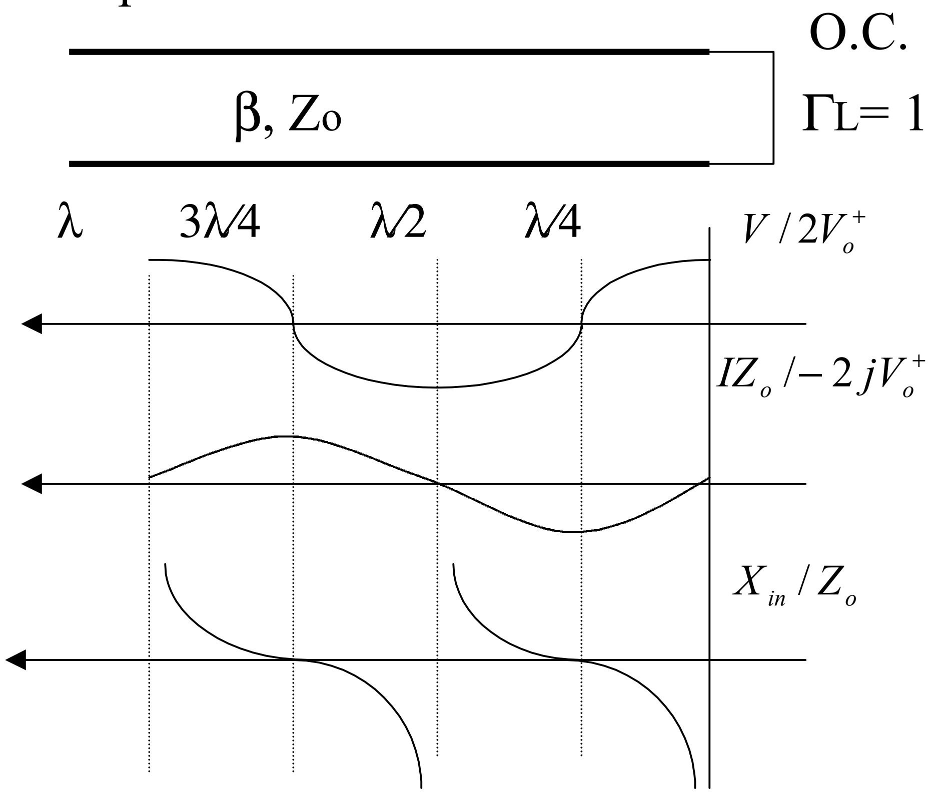
$Z_{in}(z) = Z_o \rightarrow$ no reflected wave $\Gamma(z)=0$, VSWR=1, RL= ∞dB

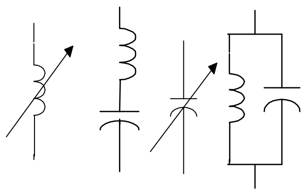
$P_{av} = P_{av,max}$: maximum power delivered to the load

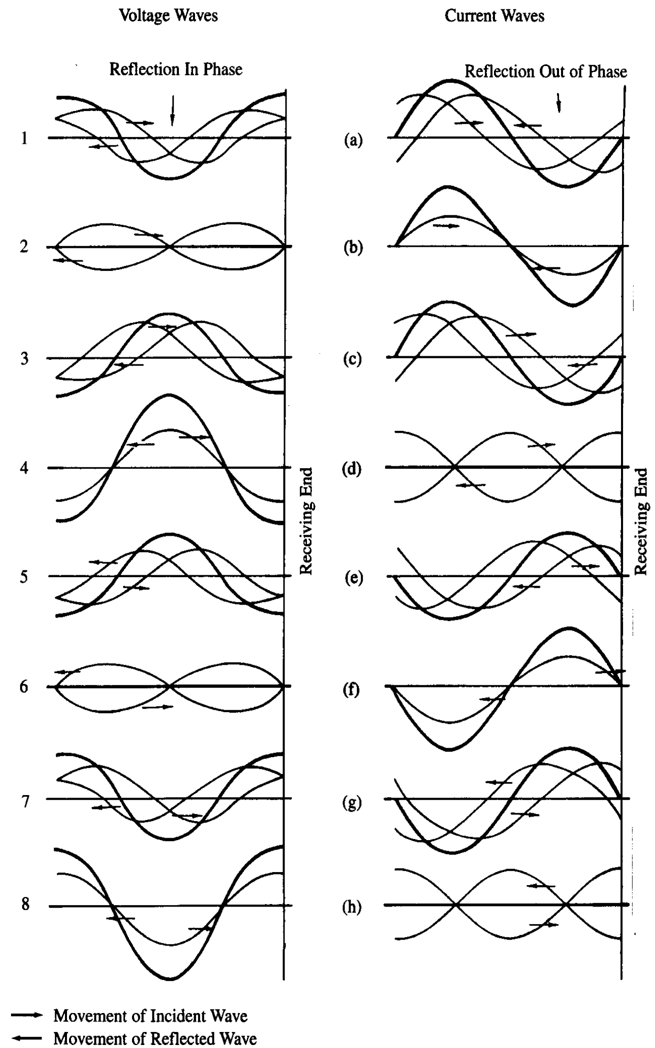


2.9 Open and shorted terminations

1.Open circuit termination







Derivation for the plots in 2-22 and 2-24

open – circuited transmission line

$$V = V_o^+ (e^{-j\beta z} + e^{j\beta z}) = 2V_o^+ \cos\beta z \quad z<0$$

$$\because l = -z, V = 2V_o^+ \cos\beta l \text{ or } \frac{V}{2V_o^+} = \cos\beta l$$

$$I = \frac{V_o^+}{Z_o}(e^{-j\beta z} - e^{j\beta z}) = -j2\frac{V_o^+}{Z_o}\sin\beta z = j2\frac{V_o^+}{Z_o}\sin\beta l \text{ or } \frac{I}{\text{-}j2V_o} = -\sin\beta l$$

$$Z_{in} = \frac{Z_0}{j\tan\beta l} = jX_{in} \text{ or } \frac{X_{in}}{Z_0} = \frac{-1}{\tan\beta l}$$

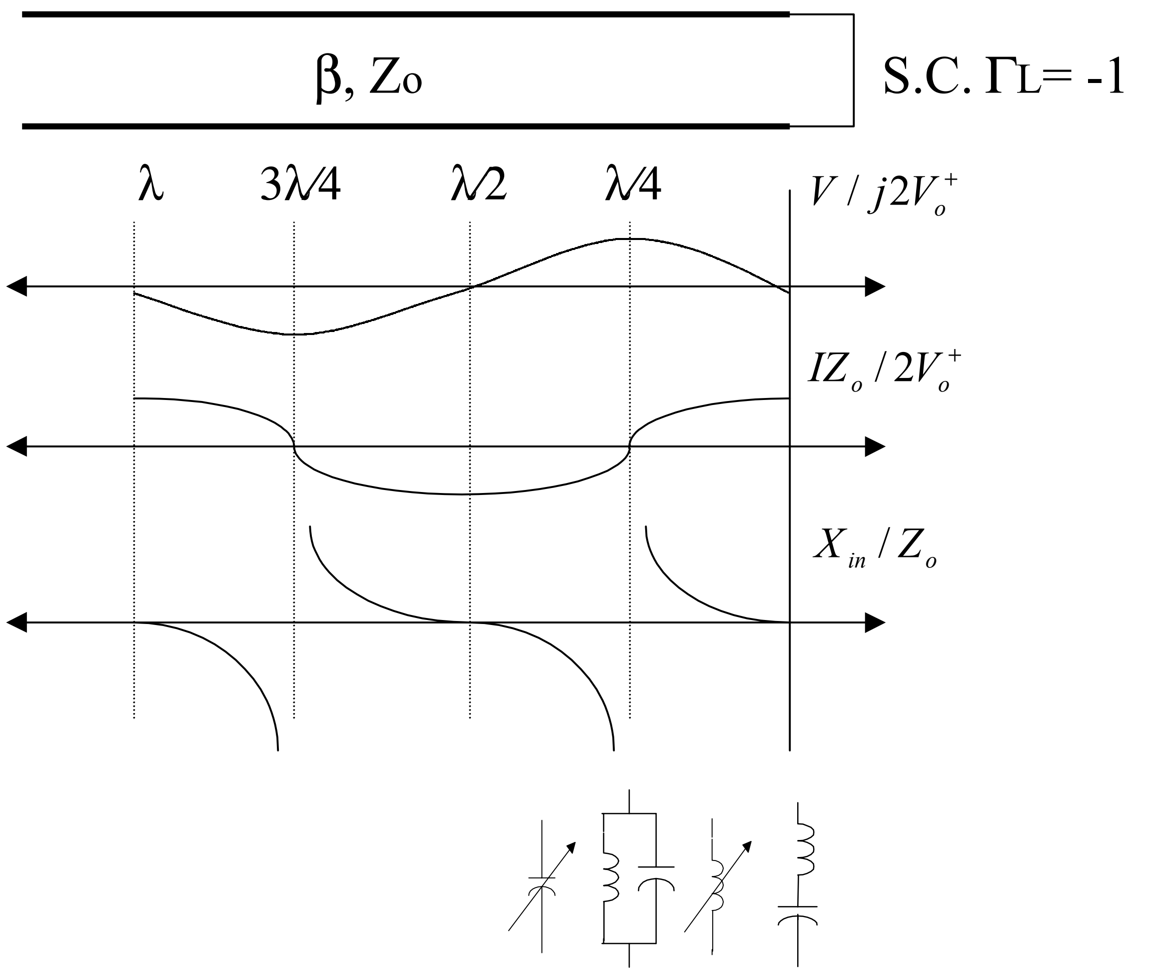
similarly, for short – circuited transmission line

$$V = V_o^+ (e^{-j\beta z} - e^{j\beta z}) = -j2V_o^+ \sin\beta z = j2V_o^+ \sin\beta l \text{ or } \frac{V}{j2V_o^+} = \sin\beta l$$

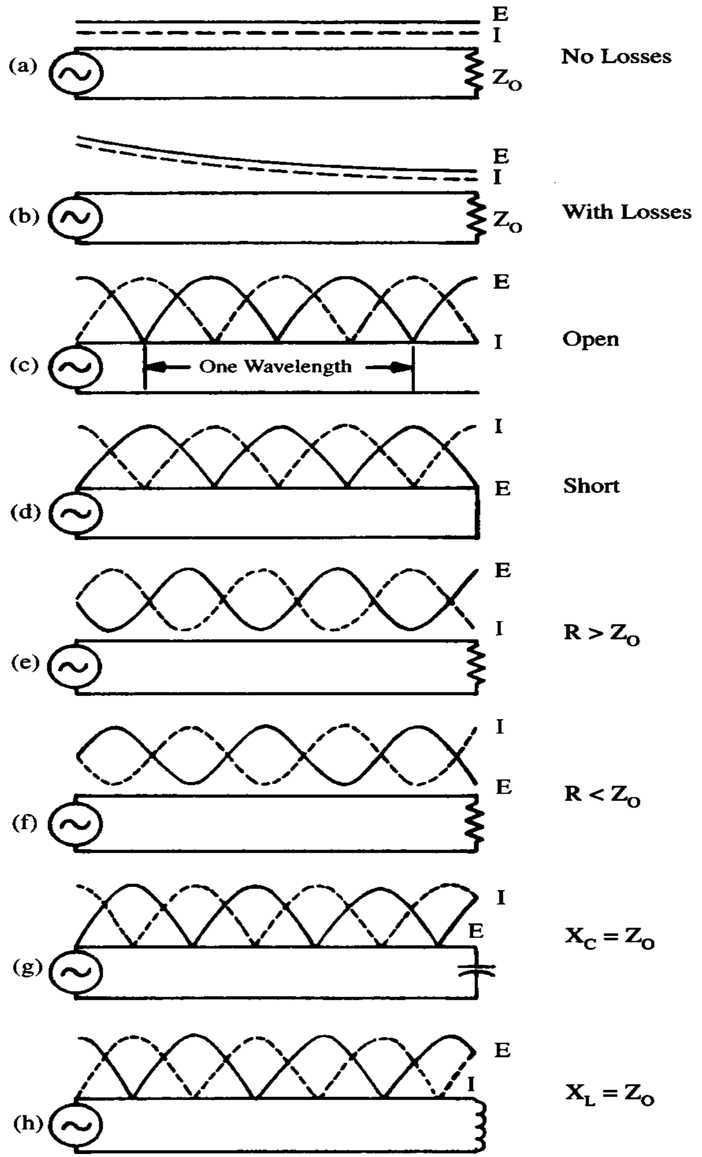
$$I = \frac{V_o^+}{Z_o}(e^{-j\beta z} + e^{j\beta z}) = 2\frac{V_o^+}{Z_o}\cos\beta z = 2\frac{V_o^+}{Z_o}\cos\beta l \text{ or } \frac{I}{2V_o} = \cos\beta l$$

$$Z_{in} = jZ_o\tan\beta l = jX_{in} \text{ or } \frac{X_{in}}{Z_0} = \tan\beta l$$

2.Short circuit termination



3. Effects of various termination on standing waves



2.10 Transmission line sections

1. quarter-wave "transformer" or impedance inverter

$$Z_{in}(l) = Z_o \frac{Z_L + jZ_o \tan \beta l}{Z_o + jZ_L \tan \beta l}$$

$l=\lambda/4$, $Z_{in}(l)=Z_o^2/Z_L$ at f_o

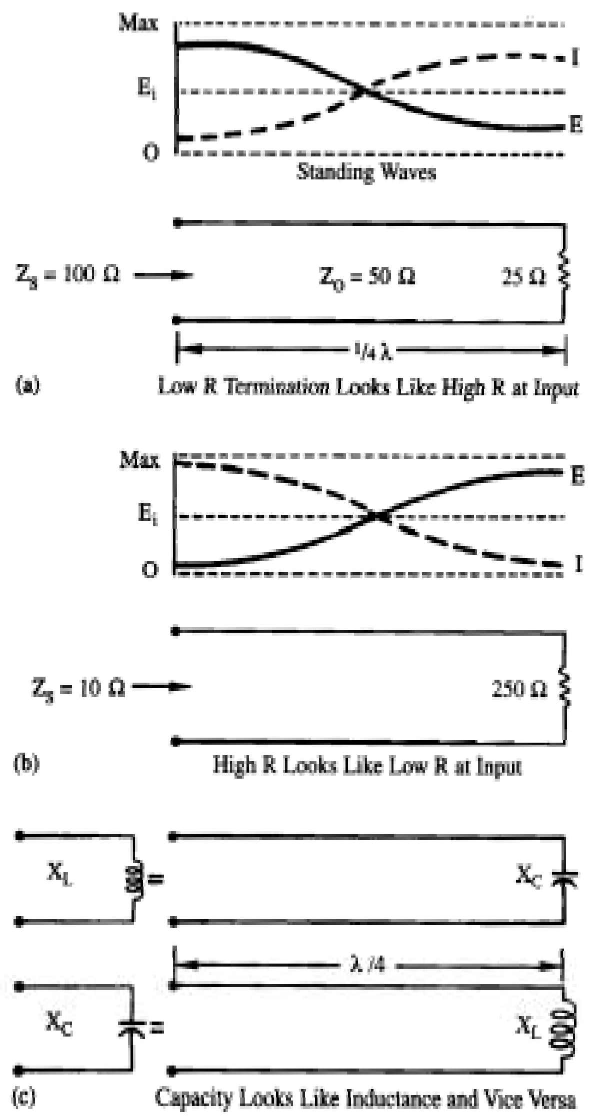
2. Ex. 2.15 Find Z_o and length of a $\lambda/4$ line (ε_r=2.25) to match a 100Ω to a 50Ω line @500MHz.

$$Z_o = \sqrt{50 \times 100} = 70.7\Omega$$

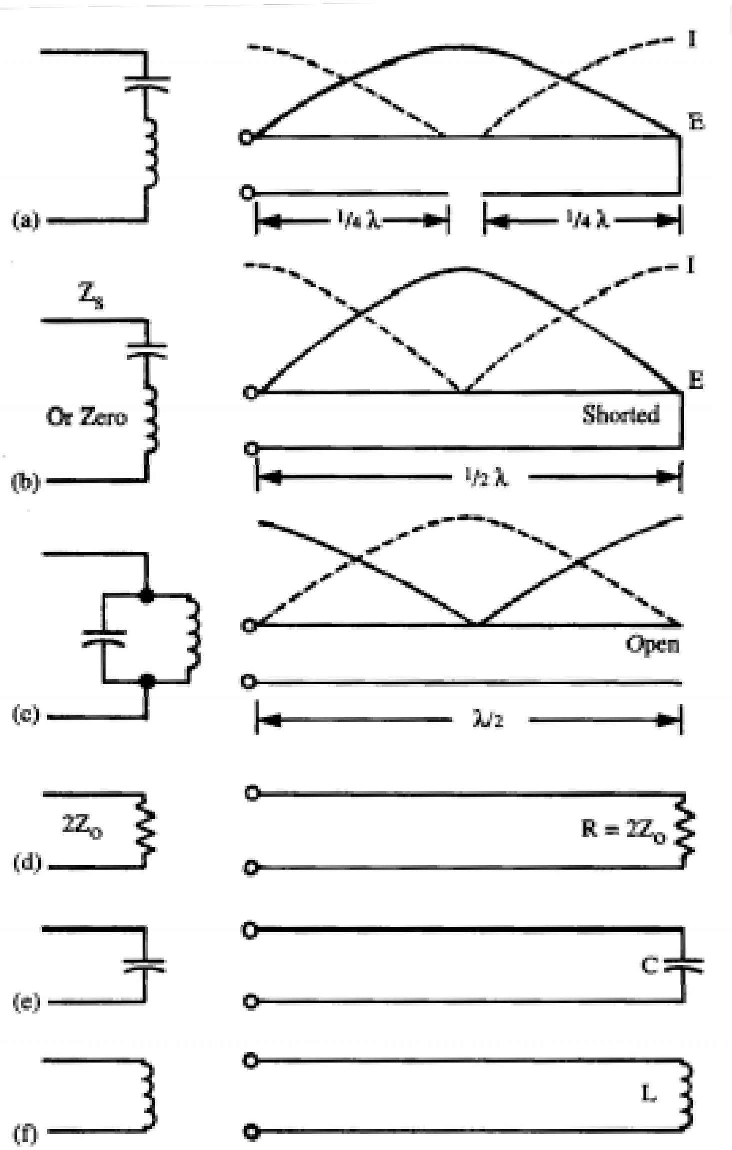
$$l = \frac{\lambda}{4} = \frac{v}{4f} = \frac{c}{4f\sqrt{\varepsilon_r}}$$

$$= \frac{3 \times 10^{10}}{4 \times 500 \times 10^{6} \times \sqrt{2.25}}$$

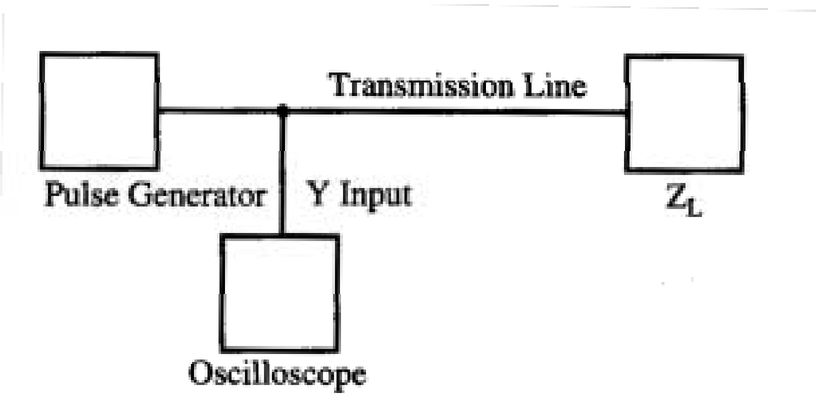
$$= 10\,cm$$

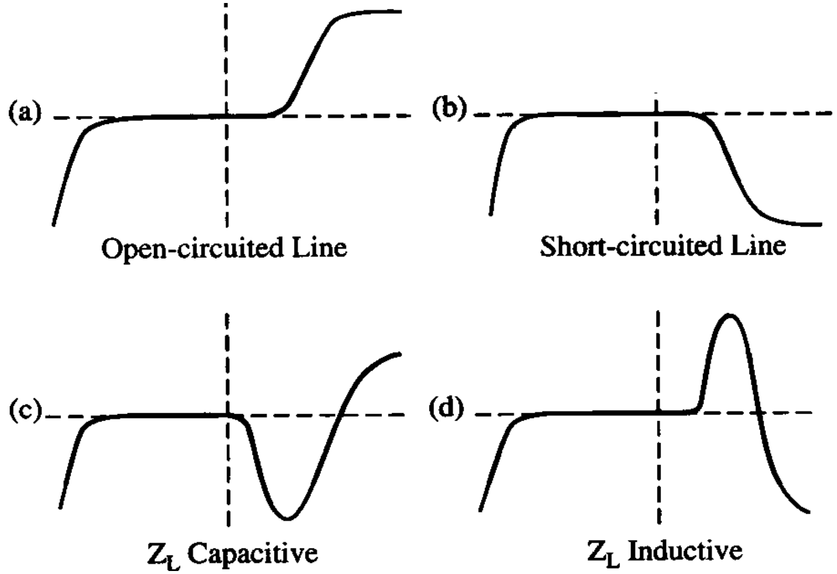


4. Half-wavelength line $l=\lambda/2$, $Z_{in}(l)=Z_L$ at f_o



2.11 Time-domain reflectometry (TDR)





1. Time of the reflection → defect position
   reflected signal → types of defect
2. Fast rise time of step pulse is required to identify the multiple faults

<u>Homework #2</u> (due 2 weeks)
Chap.2: problems 1-16