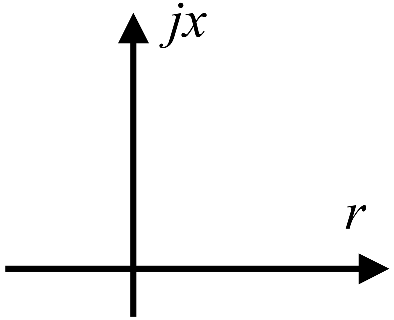
Chapter 3 Smith chart analysis

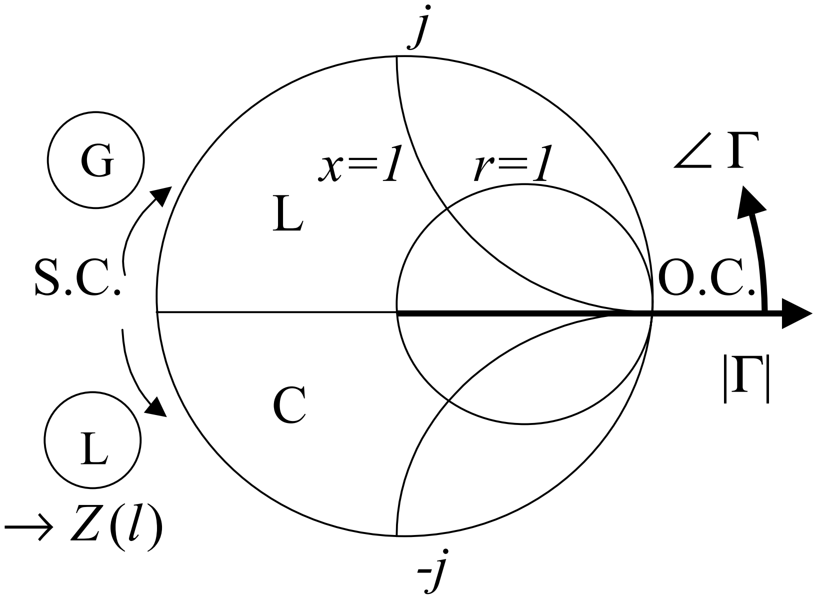
3.1 Introduction
  Smith chart ($Z \rightarrow \Gamma$) and its properties
3.2 Using the Smith chart
  7 examples
3.3 Short-circuit shift method
  find Z_L from VSWR and minima shift distance
3.4 Impedance matching techniques
  $\lambda/4$ transformer, single-stub tuner
3.5 Smith chart radially scaled parameters

3.1 Introduction

Smith chart : conformal map rectangular plot of $z = Z/Z_o = r + jx$
on the polar plot of $\Gamma = |\Gamma| e^{j\angle\Gamma} = \Gamma_r + j\Gamma_i,\ |\Gamma| \le 1,\ -180^o \le \angle\Gamma \le 180^o$

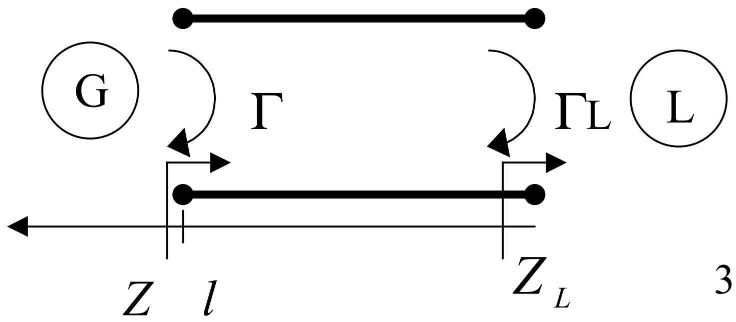


$$\Gamma = \frac{Z - Z_o}{Z + Z_o} = \frac{z-1}{z+1}$$

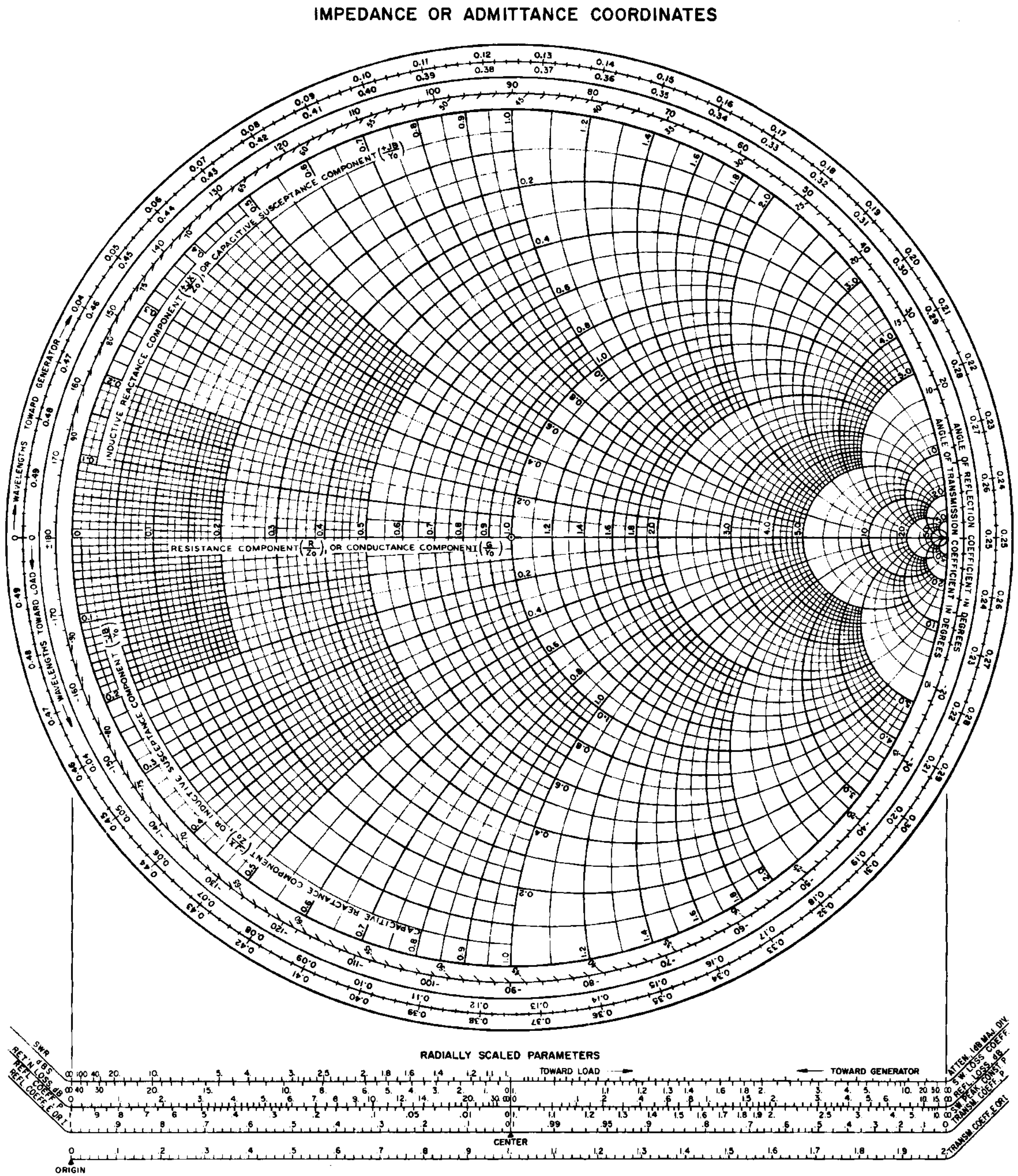


1. $Z_L \rightarrow z_L \rightarrow \Gamma_L \rightarrow \Gamma(l) \rightarrow z(l) \rightarrow Z(l)$

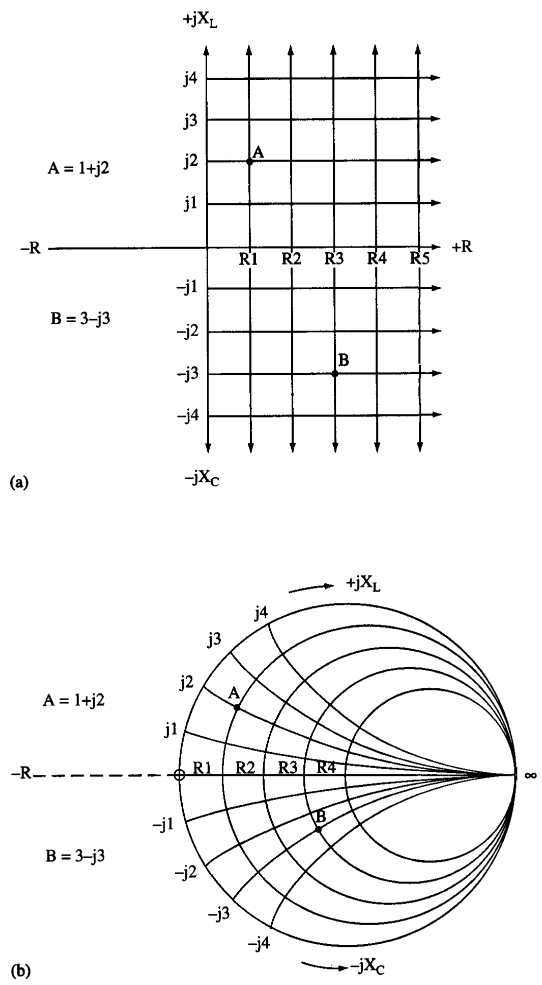
toward G : CCW



$$\Gamma(l) \equiv \frac{V^-(l)}{V^+(l)} = \frac{V_o^- e^{-j\beta l}}{V_o^+ e^{j\beta l}} = \Gamma_L e^{-j2\beta l}$$



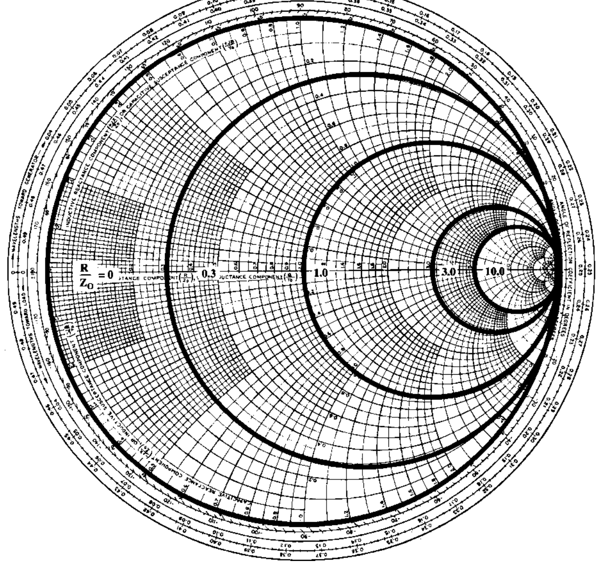
IMPEDANCE OR ADMITTANCE COORDINATES
RADIALLY SCALED PARAMETERS
TOWARD LOAD
TOWARD GENERATOR
CENTER
ORIGIN



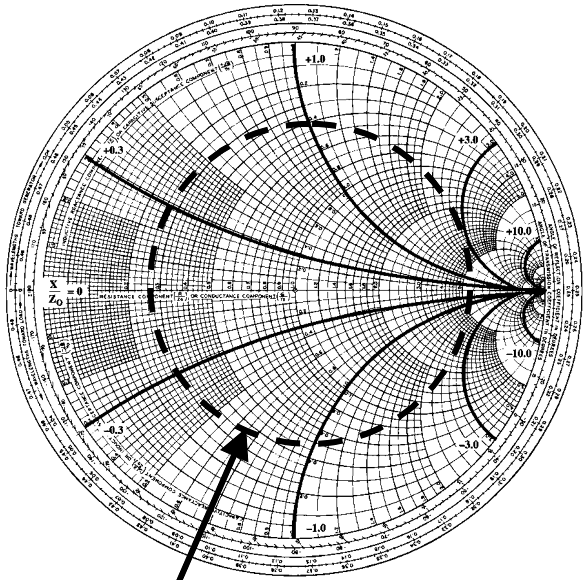
+jX_L
A = 1+j2
B = 3–j3
–R
+R
R1 R2 R3 R4 R5
j4 j3 j2 j1 –j1 –j2 –j3 –j4
–jX_C
(a)
A = 1+j2
B = 3–j3
R1 R2 R3 R4
∞
(b)

微波工程講義

2. r - circle $\perp$ x - circle



constant r-circle



constant x-circle

constant |Γ|-circle or
constant VSWR-circle

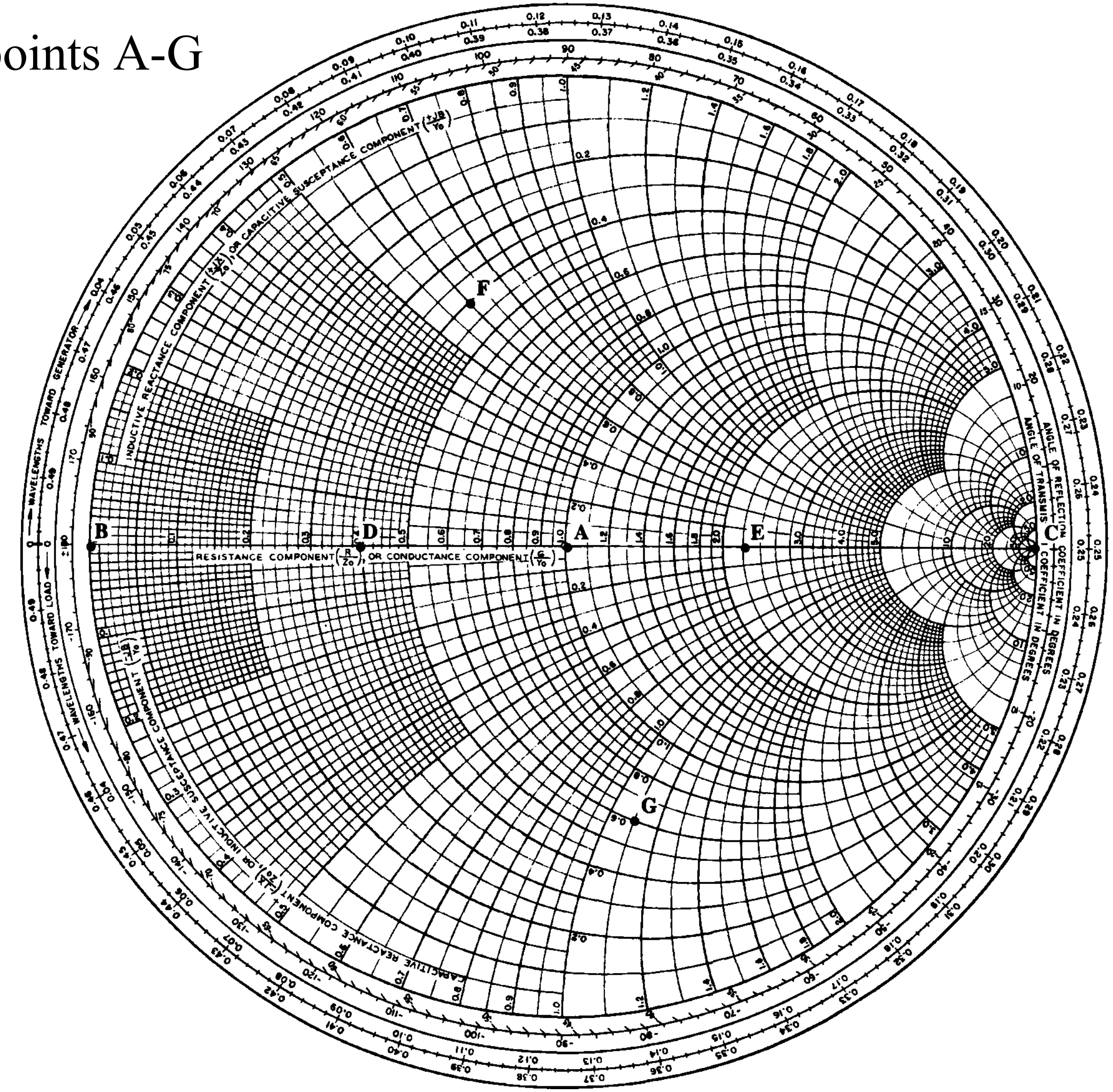
3. $VSWR = r_{\max} = \dfrac{R_{\max}}{Z_o} = \dfrac{1}{r_{\min}} = \dfrac{Z_o}{R_{\min}}$

4. $\mathrm{y} = z^{-1} \rightarrow \angle\Gamma + 180^o$

$$\because \Gamma = \frac{Z - Z_o}{Z + Z_o} = \frac{z-1}{z+1}, \Gamma' = \frac{z^{-1}-1}{z^{-1}+1} = \frac{1-z}{1+z} = -\Gamma$$

5. $l = \lambda / 2 \rightarrow 360^o$

6. Ex. points A-G



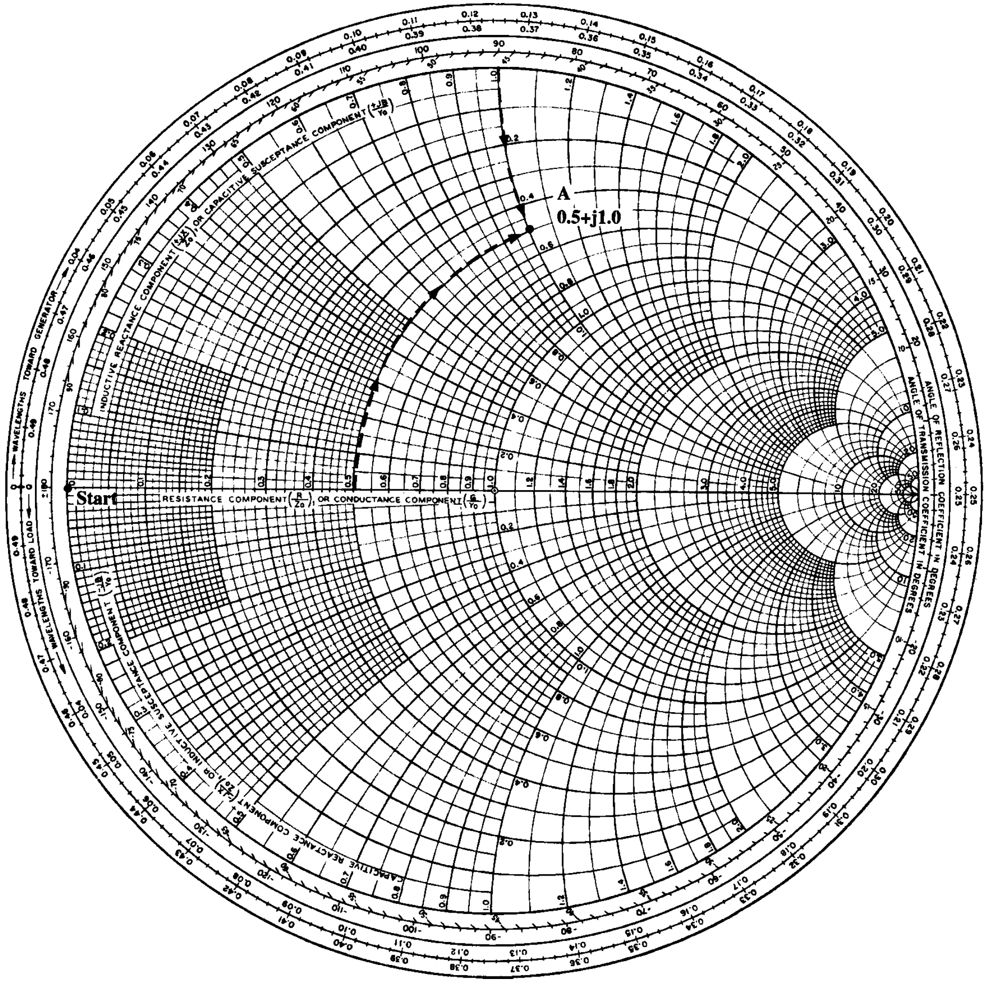
3.2 Using the Smith chart

1. Examples to solve the transmission line problems:

- Ex.1 plot load impedance Z_L
- Ex. 2, 3, 4, 5, 7
  given $Z_L \rightarrow$ determine Γ, VSWR, Z_{in}, T (transmission coefficient), RL, Y, Z_{max} and Z_{min} on the line
- Ex. 4
  given $Z_L \rightarrow$ determine the distance to the max. and min. points of a standing wave
- Ex.8 given VSWR, short-circuit minima $\rightarrow Z_L$
- Ex.6 given $Z_L \rightarrow$ determine the relative λ position of load
- given a distance from the load $\rightarrow$ determine the line impedance
- Ex.9 given $Z_L \rightarrow$ design a $\lambda/4$ transformer
- Ex.10, 11 given $Z_L \rightarrow$ design single- and double-stub tuners

2. Ex. 3.1 Z_o=50Ω, plot Z_L=25+j50Ω

z_L=0.5+j1 : A



3. Ex. 3.2 Z_o=50Ω, Z_L=50+j50Ω find VSWR

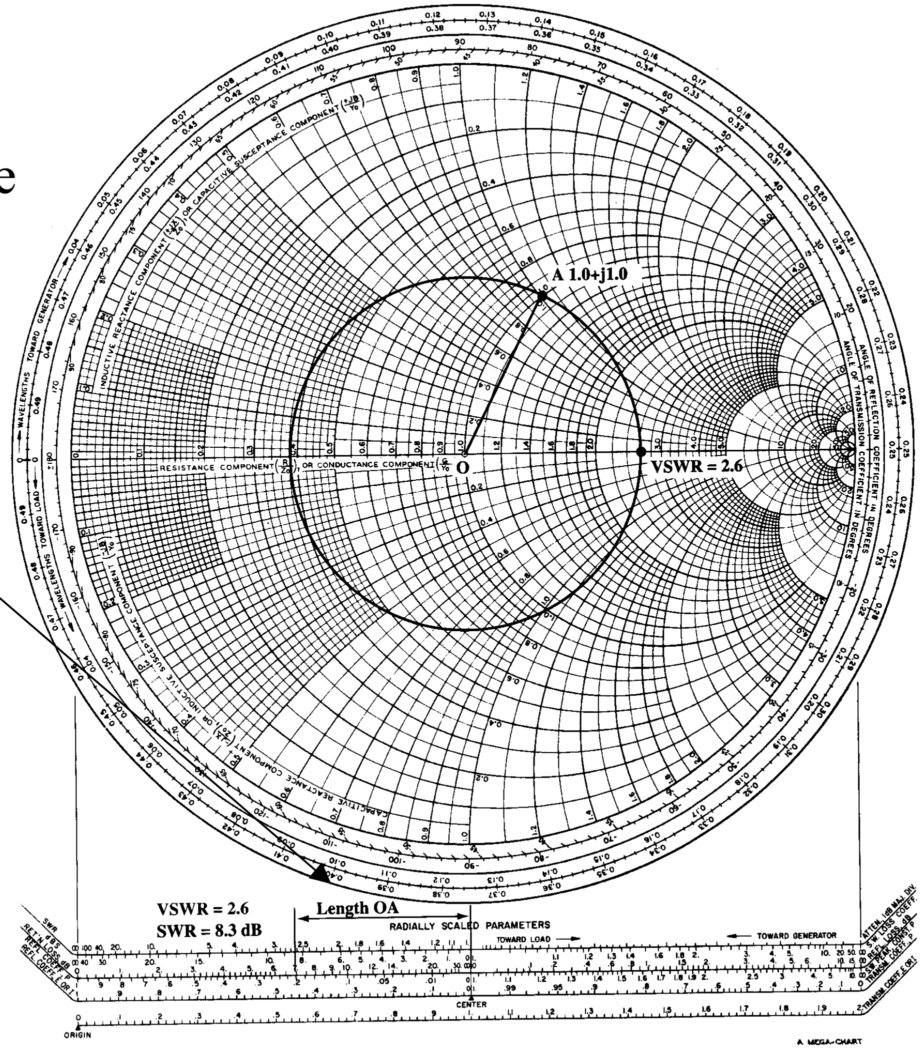
1) z_L=1+j1 : A

2) draw constant VSWR-circle

3) read r_{max}=2.6=VSWR

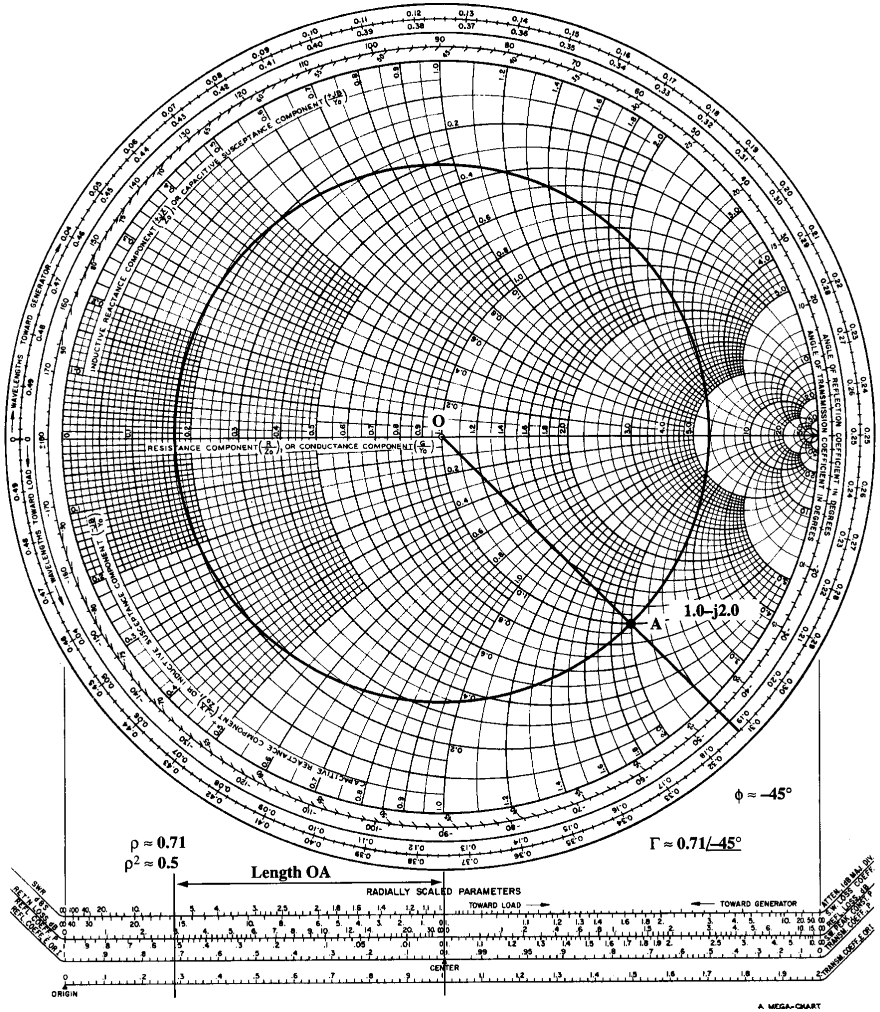
or

read distance OA from radially scaled parameters



4. Ex. 3.3 Z_o=100Ω, Z_L=100-j200Ω find Γ

1) z_L=1-j2: A
2) draw constant VSWR-circle
3) read distance OA from radially scaled parameters $|\Gamma|$= 0.71
4) read phase -45°
5) read power reflection coefficient 0.5 = $|\Gamma|^2$

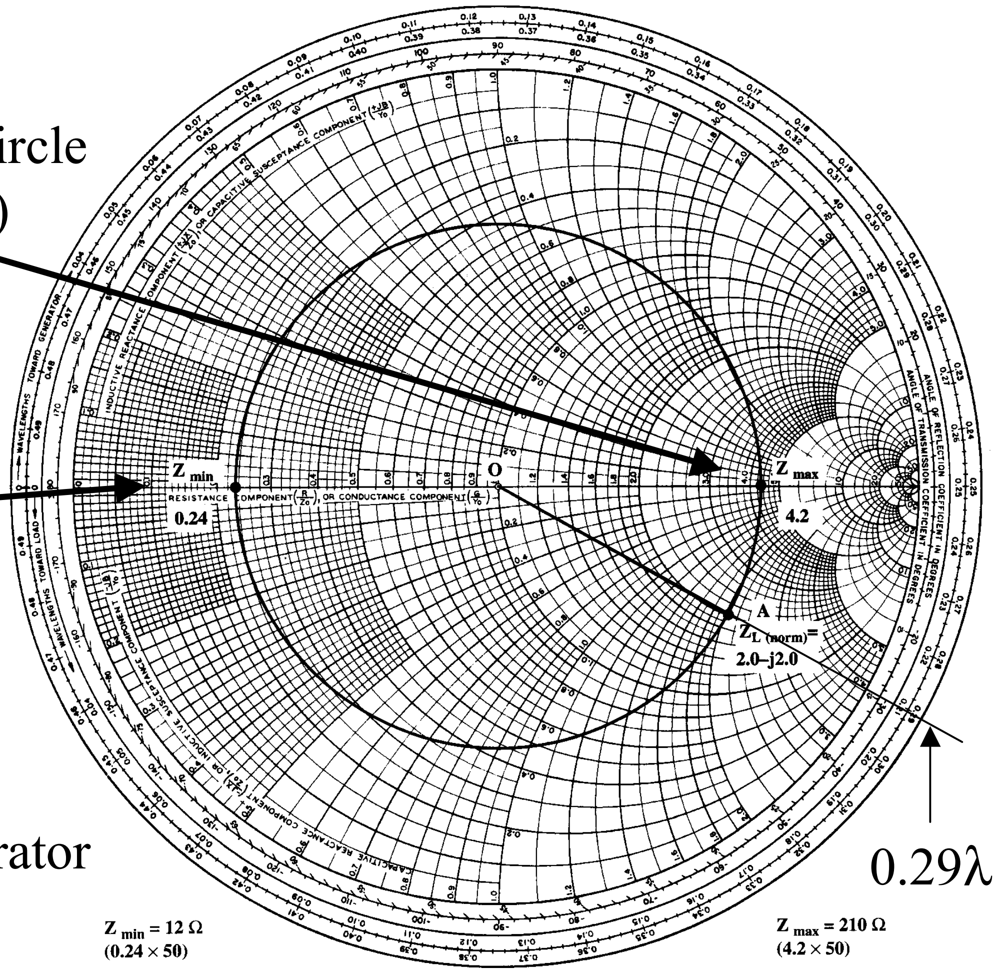


reflection coefficient P

reflection coefficient E or I

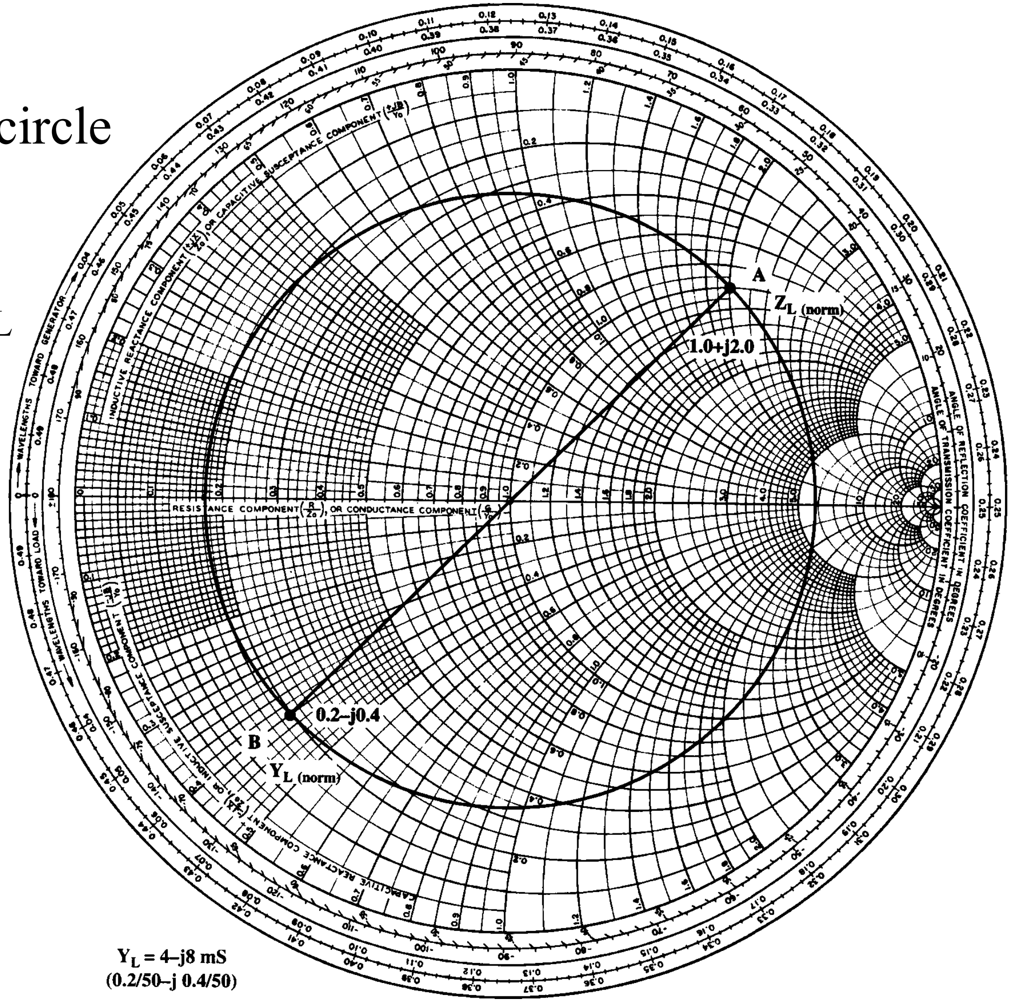
5. Ex. 3.4 Z_o=50Ω, Z_L=100-j100 Ω find Z_{max} and Z_{min} on the line

1) z_L=2-j2: A
2) draw constant VSWR-circle
3) read z_{max}=4.2 (=VSWR)
   → Z_{max}=210 Ω
   at 0.04λ toward load direction
   or at 0.46λ toward generator direction
4) read z_{min}=0.24 or 1/4.2
   → Z_{min}=12 Ω
   at 0.21λ toward generator
   or at 0.29λ toward generator direction



6. Ex. 3.5 Z_o=50Ω, Z_L=50+j100 Ω find Y_L

1) z_L=1+j2: A
2) draw constant VSWR-circle
3) read B (λ/4 away)
   y_L=0.2-j0.4
4) $y_L = 1/z_L = Z_o/Z_L = Z_o Y_L$
   $\rightarrow Y_L = y_L/Z_o$
   = 0.004-j0.008S

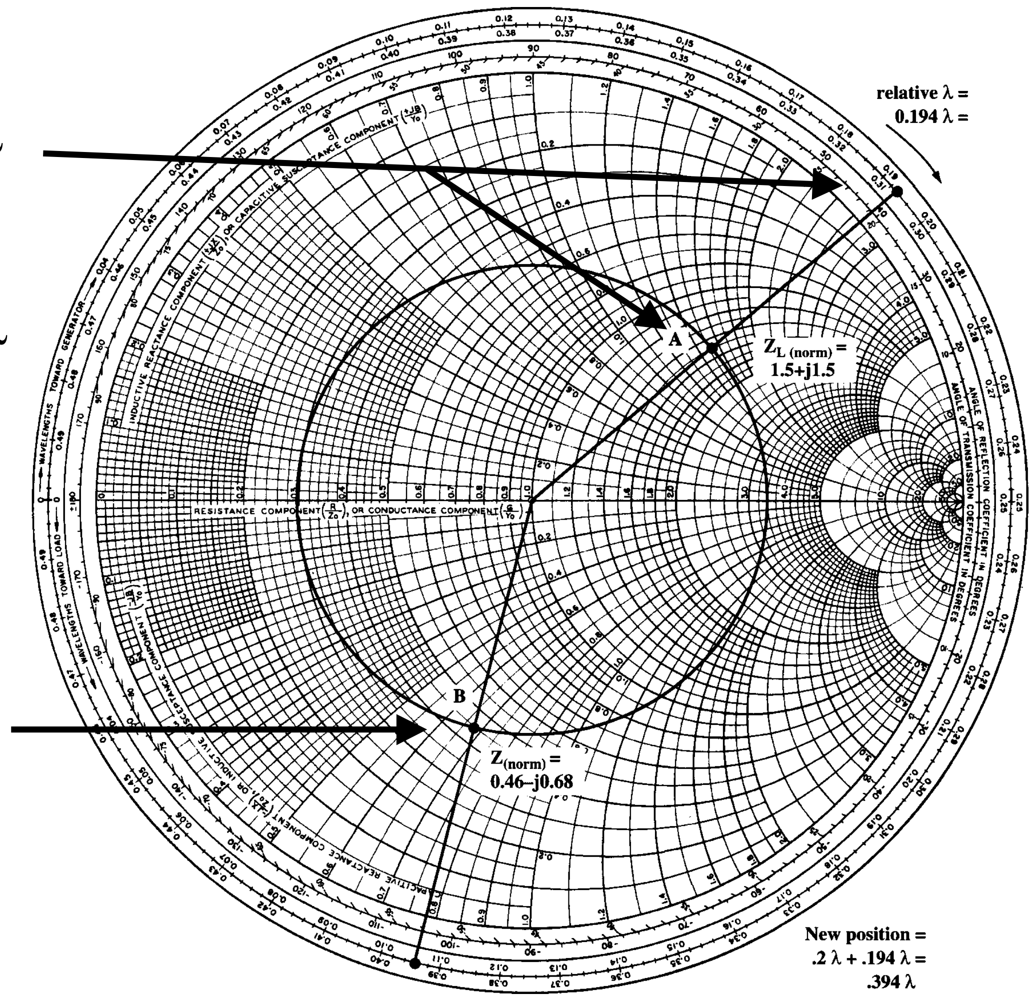


7. Ex. 3.6 Z_o=50Ω, Z_L=75+j75 Ω find Z_L relative location

1) z_L=1.5+j1.5: A
2) read location at 0.194λ toward generator

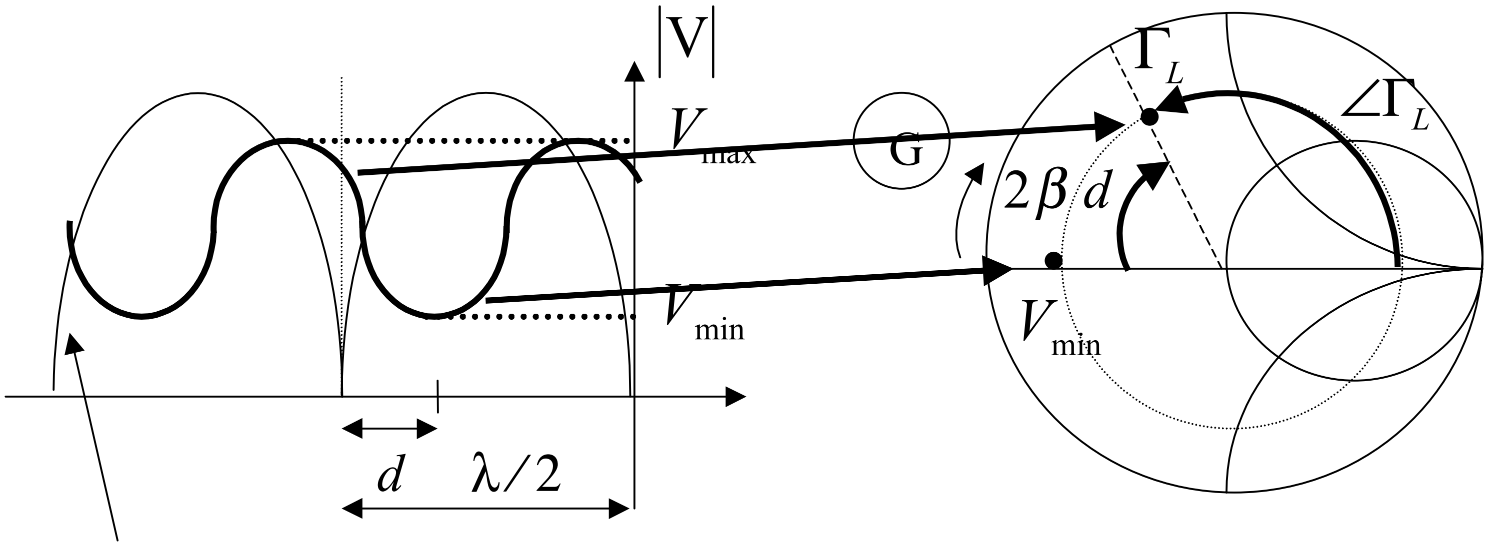
8. Ex. 3.7 Find Z_{in} at 0.2λ from the load

1) draw constant VSWR circle
2) move 0.2λ toward generator (CW)
3) read B: z = 0.46-j0.68
→ Z_{in}=34.5-j51Ω



3.3 Short-circuit minima shift method

1. given $VSWR$, distance minimum shifted $d \Rightarrow Z_L$



short-circuit load
VSWR pattern

$$VSWR = \frac{1+|\Gamma_L|}{1+|\Gamma_L|} \rightarrow |\Gamma_{\mathrm{L}}| = \frac{VSWR-1}{VWSR+1}$$

$$180^o - 2\beta d = 180^o(1-4d/\lambda) = \angle\Gamma_{\mathrm{L}}$$

2. Ex. 3.8 $Z_o=50\Omega$, VSWR=2, short-circuit minima at 36.9mm and 55.2mm, load minima shifted to 42.9mm, find Z_L

$\lambda=2(55.2-36.9)=36.6\text{mm}$

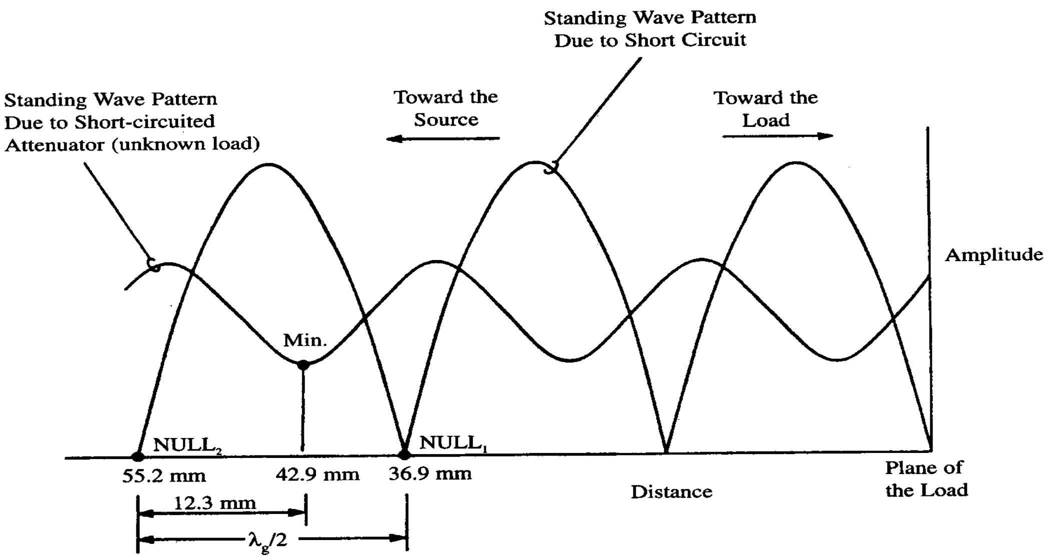
$d=55.2-42.9=12.3\text{mm}$

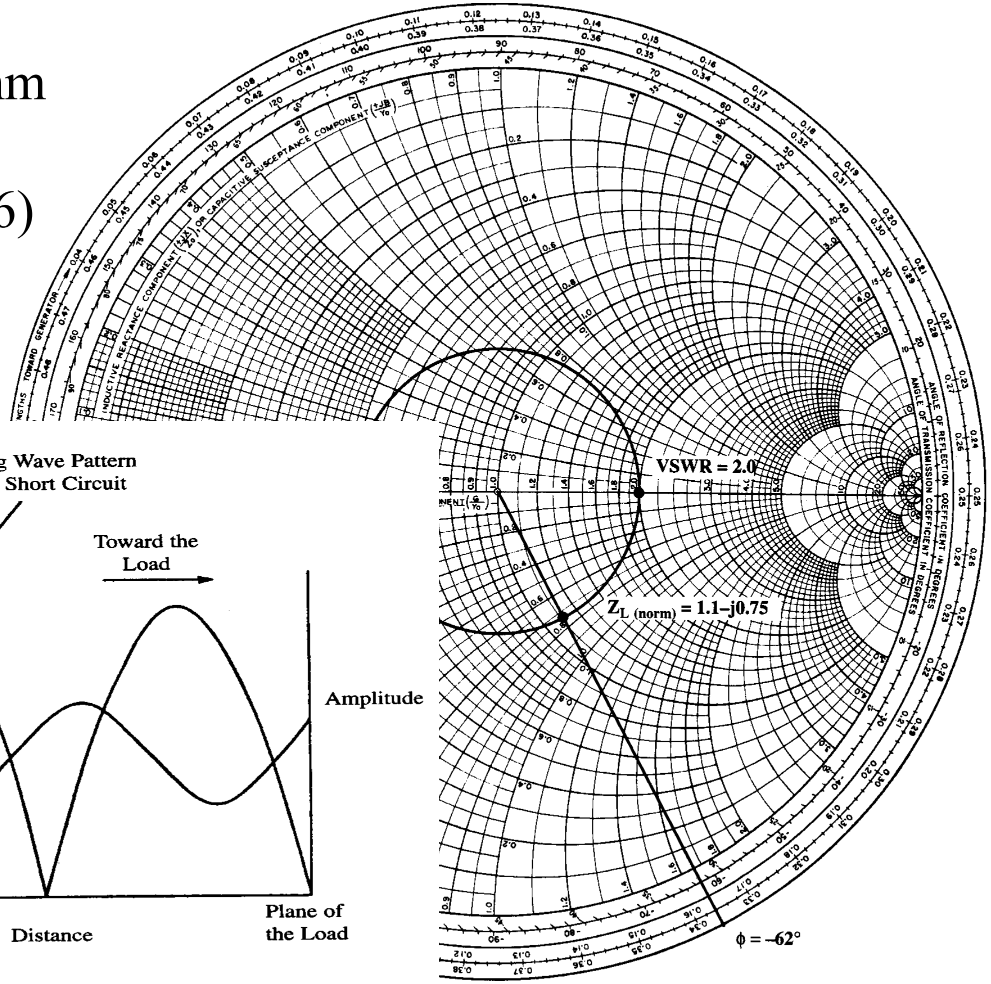
$\angle\Gamma=180(1-4\times12.3/36.6)$

$\quad=-62^\circ$

$z_L=1.1-j0.75$

$Z_L=55-j37.5\Omega$

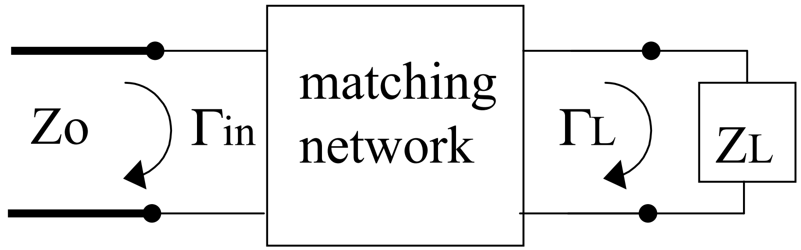




3.4 Impedance matching technique

1. Impedance matching concept

given Z_L, design a matching network to have $\Gamma_{in} = 0$ or selected Γ_{in} value

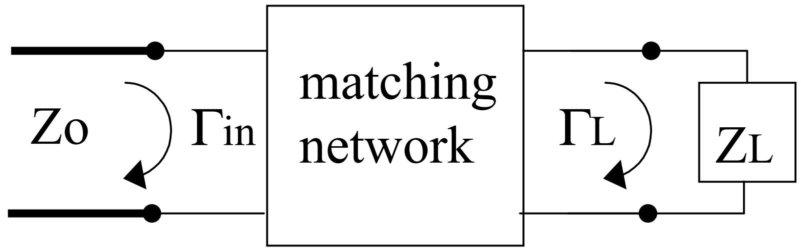


2. Matching network usually uses lossless elements: L, C, transmission line, transformer,…for a narrow bandwidth match.
3. Lossy matching elements, e.g. resistive pad, can give a broadband match, but degrades the circuit efficiency.
4. There are ∞ possible solutions.
5. Use Smith chart to find the optimal design.

3.4 Impedance matching technique

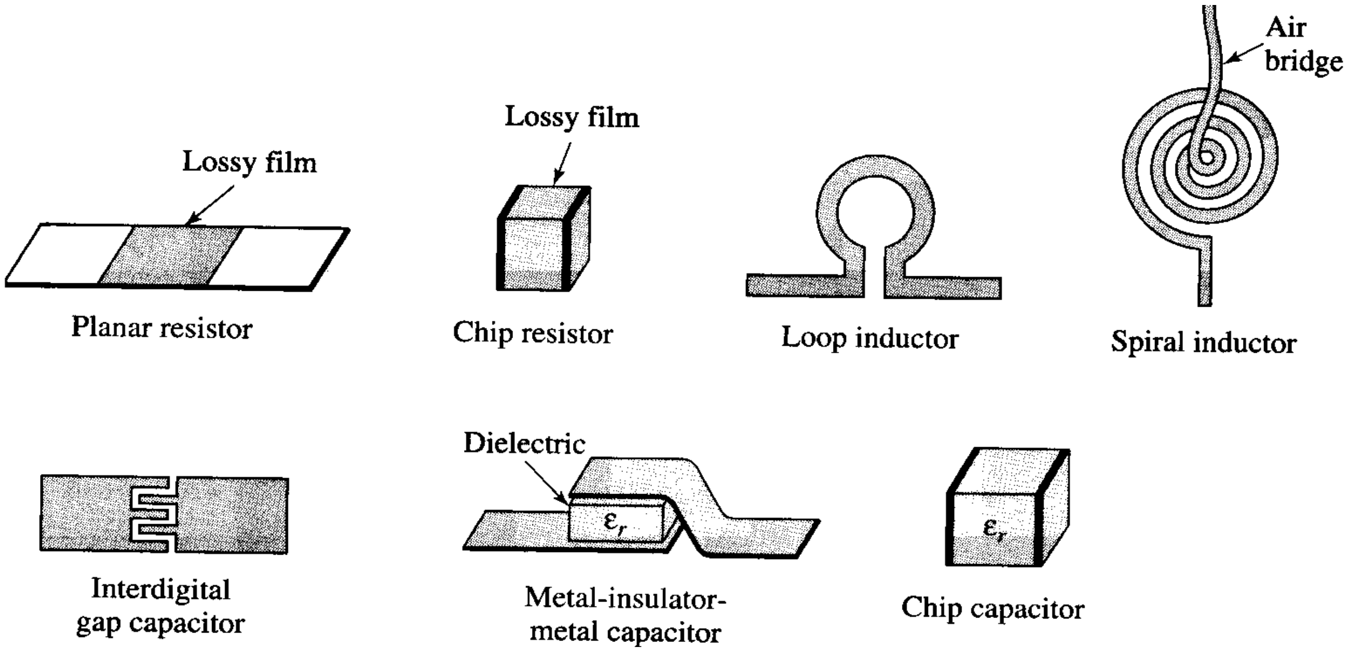
1. Impedance matching concept

   given Z_L, design a matching network to have $\Gamma_{in} = 0$ or selected Γ_{in} value

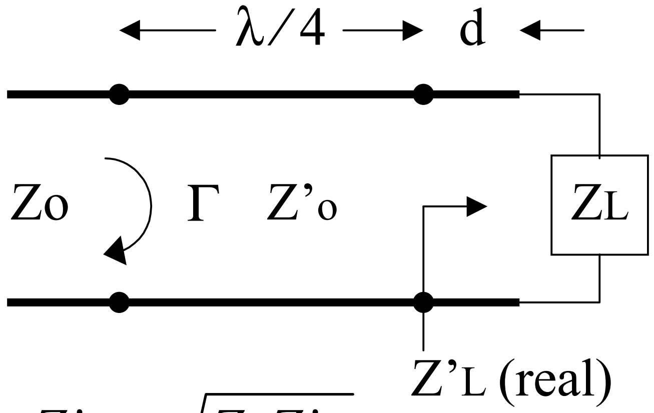


2. Matching network usually uses lossless elements: L, C, transmission line, transformer,…for a narrow bandwidth match.
3. Lossy matching elements, e.g. resistive pad, can give a broadband match, but degrades the circuit efficiency.
4. There are ∞ possible solutions.
5. Use Smith chart to find the optimal design.

6. Factors for selecting matching circuit: complexity, bandwidth, implementation and adjustibility
7. Lumped elements (size<λ/10) for microwave circuits
   capacitor: chip capacitor, MIM capacitor (<25pF), interdigital
   gap capacitor (<0.5pF), open stub(<0.1pF)
   inductor: chip inductor, loop inductor, spiral inductor (<10nH)
   resistor: chip resistor, planar resistor

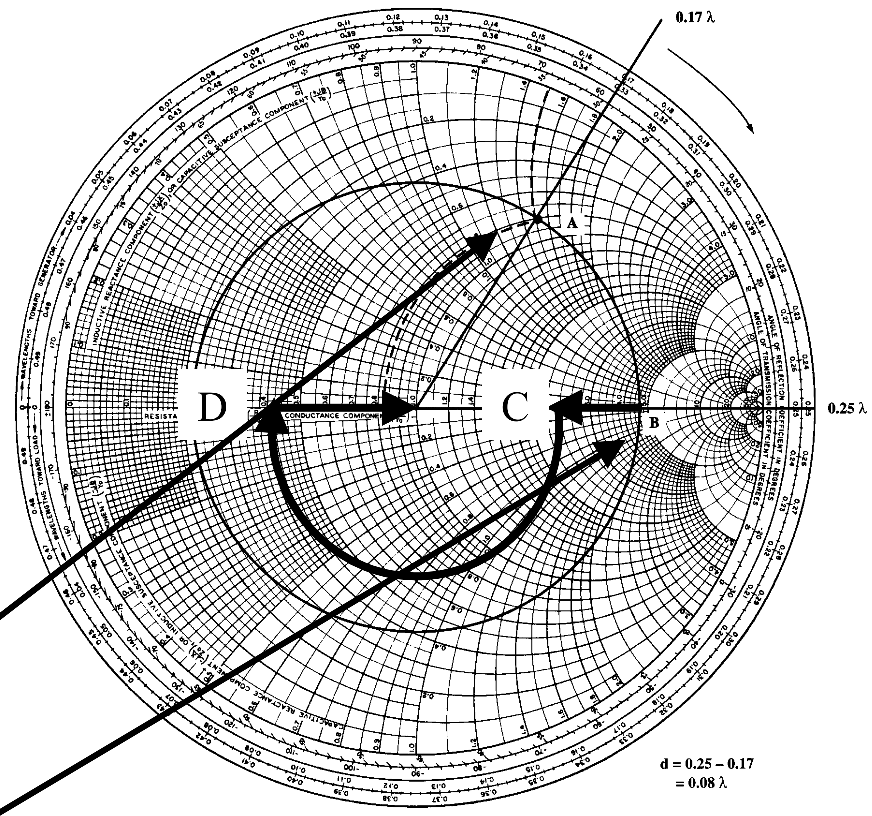


8. $\lambda/4$ transformer



$$Z'_o = \sqrt{Z_o Z'_L}$$

9. Ex. 3.9 Z_o=300Ω, Z_L=250+j450 Ω
1) z_L=0.83+j1.5 : A
2) draw constant Γ-circle
3) move toward generator
B: r=4.75, d=0.08λ
4) $Z'_o = \sqrt{300 \times 4.75 \times 300}$
$= 654\Omega$

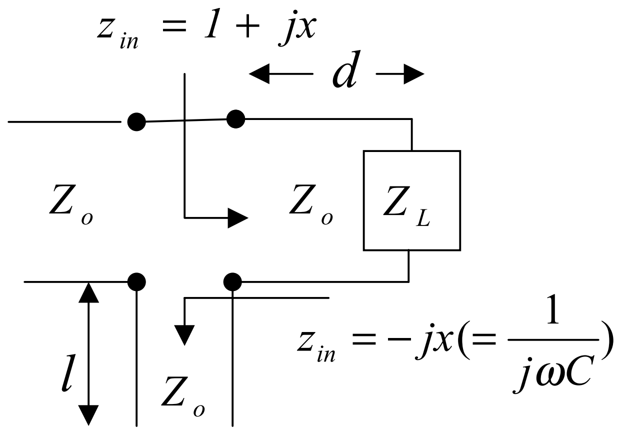


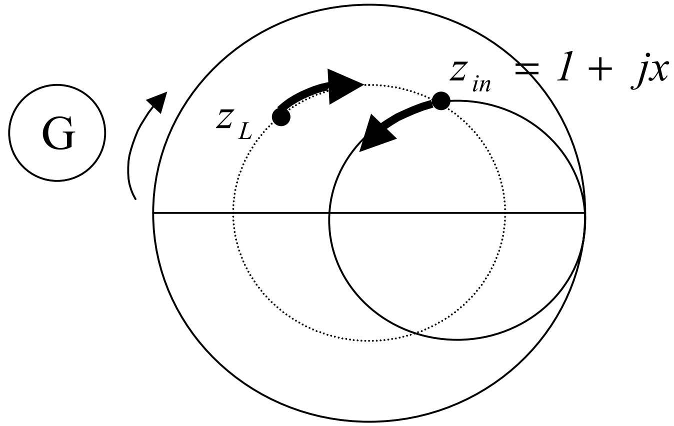
5) normalize to 654Ω
4.75×300/654=2.18 : C
moves 180° CW to D: 0.46
normalize to 300Ω
0.46×654/300=1

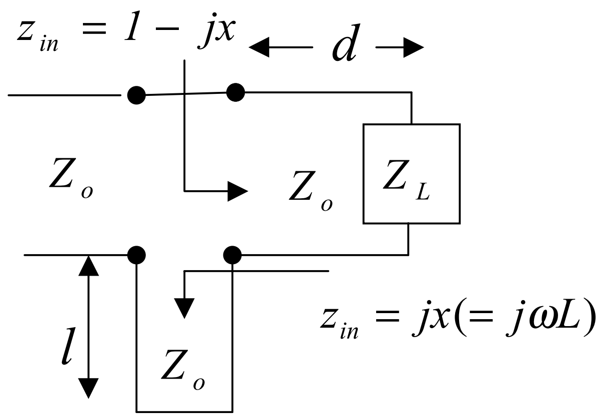
10. open-circuit stub $Z_{in} = \dfrac{Z_o}{j\tan\beta l} \equiv \dfrac{1}{j\omega C}$

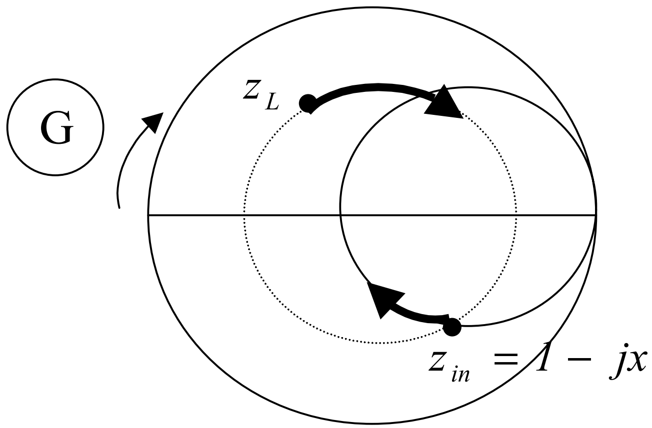
short-circuit stub $Z_{in} = jZ_o \tan\beta l \equiv j\omega L$

11. Series stub

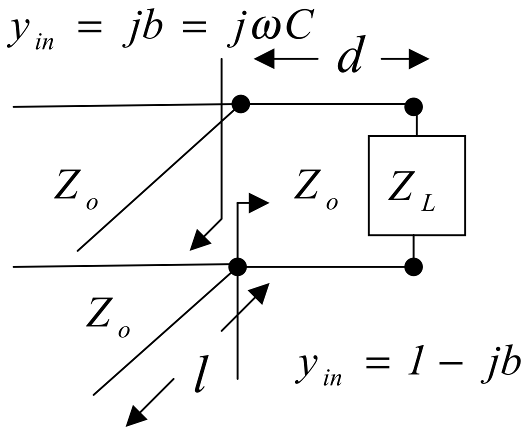


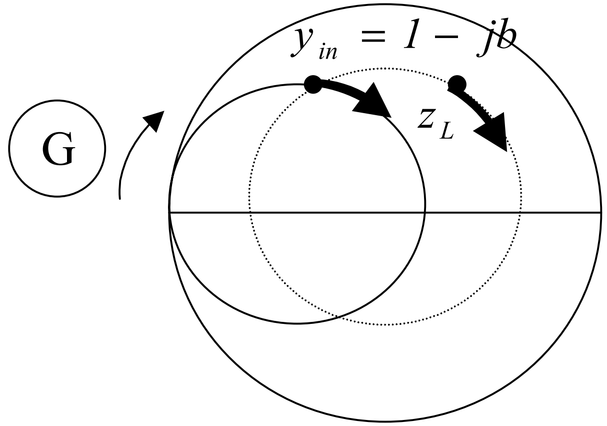


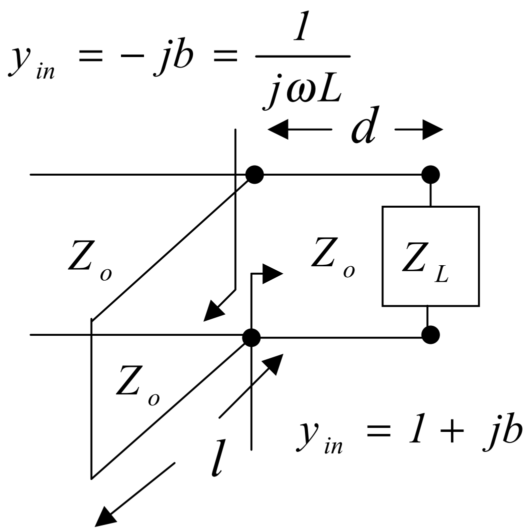


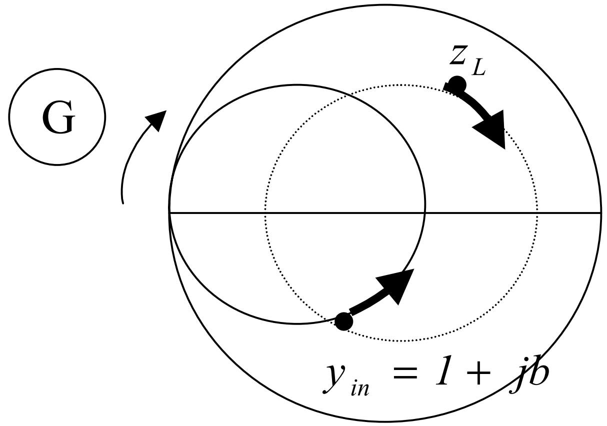


12. Shunt stub

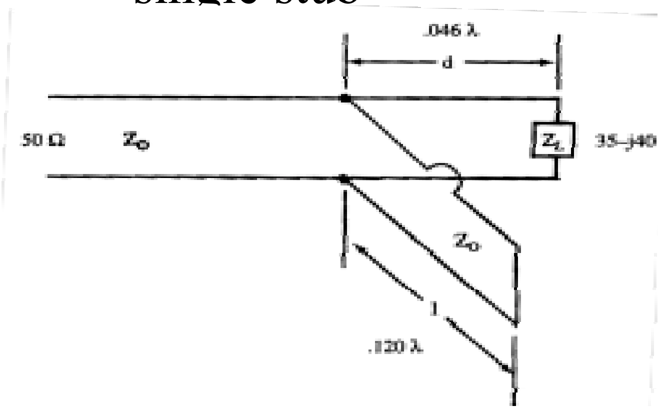


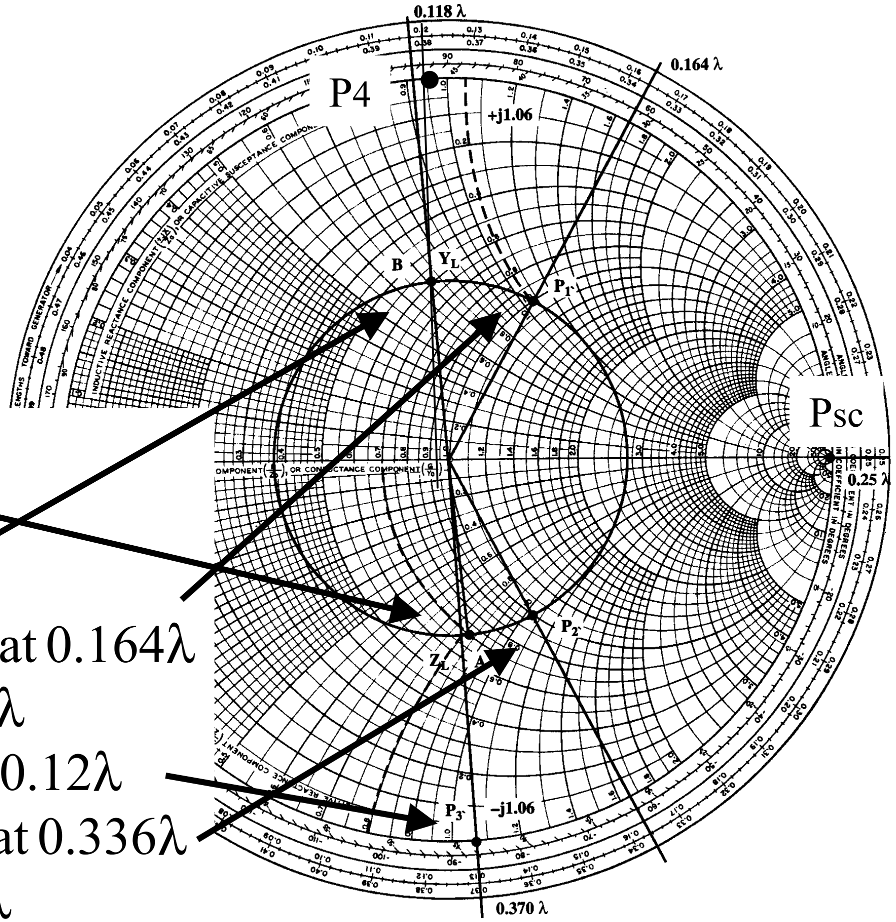






13. Ex. 3.10 Z_o=50Ω, Z_L=35-j40 Ω, find d and *l* for a short-circuit single stub

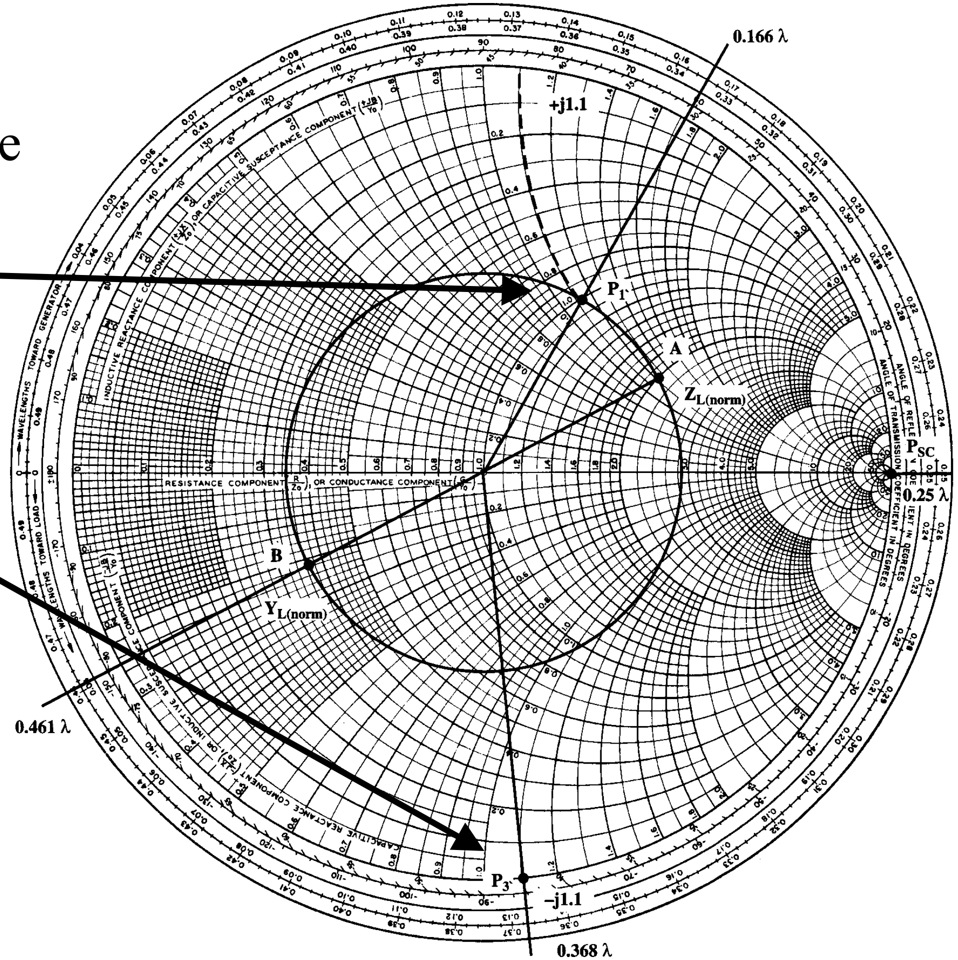




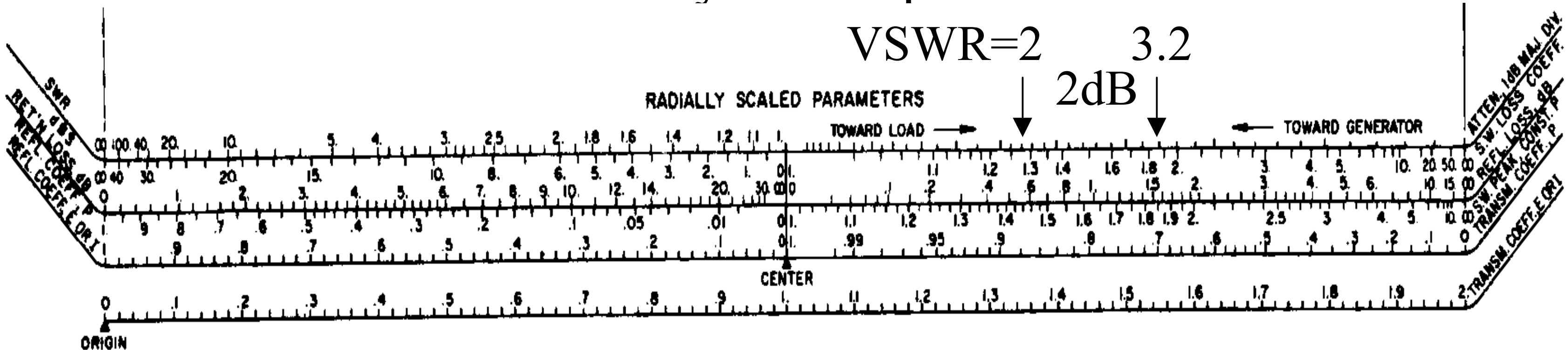
1) z_L=0.7-j0.8 :A
2) draw constant VSWR circle
3) locate y_L : B at 0.118λ
4) move CW to P_1 : y=1+j1.06 at 0.164λ
→ d_1=0.164λ-0.118λ= 0.046λ
→ P_3 : -j1.06 at 0.37λ → l_1= 0.12λ
5) move CW to P_2 : y=1-j1.06 at 0.336λ
→ d_2=0.336λ-0.118λ=0.218λ
→ P_4 : j1.06 at 0.12λ → l_2= 0.37λ
→ longer line → narrower bandwidth

14. Ex. 3.11 Z_o=50Ω, Z_L=100+j60 Ω, find d and *l* for a short-circuit single stub at 1GHz with v_f=0.95.

1) z_L=2+j1.2 : A
2) draw constant VSWR circle
3) locate y_L : B at 0.461λ
4) move CW to P_1 : 1+j1.1 at 0.166 λ
 → d= 0.5λ-0.461λ+0.166λ = 0.205λ
 → P_3 : -j1.1 at 0.368λ
 → *l*= 0.368λ-0.25λ = 0.118λ
5) λ=0.95×c/1GHz=28.5cm
 d=5.8cm, l=3.36cm



3.5 Smith chart radially scaled parameters



1. reflection: REFL. COEF. E or I $0 \leq |\Gamma| \leq 1$, P $0 \leq |\Gamma|^2 \leq 1$
   RET'N LOSS (dB) $0 \leq -20\log|\Gamma| \leq \infty$
   REFL. LOSS (dB) $0 \leq -10\log(1-|\Gamma|^2) \leq \infty$
2. transmission loss: TRANSM. COEF. E or I, P, S.W. LOSS COEF., ATTEN. 1dB for lossy line
3. SWR: $1 \leq$ VSWR $\leq \infty$, $0 \leq$ VSWR(dB) $\leq \infty$
4. Ex. A lossy 50 Ω line has 2dB attenuation. If VSWR=2 at input, find VSWR at load end.
   $VSWR = 2 \rightarrow \text{ATTEN} + 2\text{dB}$
   $\rightarrow VSWR = 3.2$

Homework #3 (due 2 weeks)
Chap.3: problems 1-14